

# Notes VI

## Brittle material behaviour

Fall 2005

### 1 Reading assignment

Twiss and Moore's chapter nine is essential reading for this stuff, and parts of chapter ten are definitely relevant.

### 2 Failure criteria

#### 2.1 Tensile stress

$$\sigma_1 = \sigma_2 >> \sigma_3$$

And  $\sigma_3$  is negative. Open tension fractures containing  $\sigma_1$  and  $\sigma_2$  and perpendicular to  $\sigma_3$ .

Draw a Mohr circle for this situation:

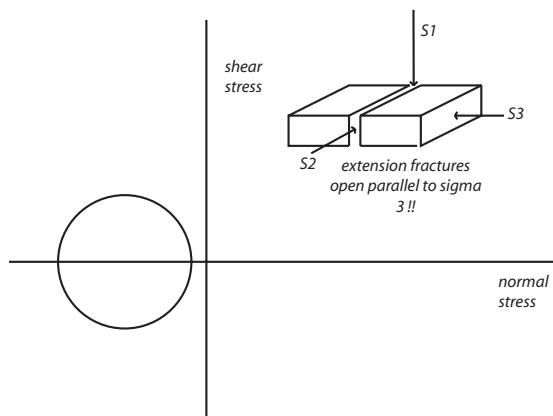


Figure 1: Mohr construction for tensile stresses

#### 2.2 Shear fractures in non-cohesive material

For instance, loose sand. Byerlee in the 50s conducted various experiments looking at the strength of these sorts of materials,

where strength is understood as the amount of shear stress necessary to initiate motion, given a certain amount of normal stress. If you plot shear stress necessary to initiate motion against normal stress, you get a line which makes an angle  $\phi$  with the normal stress axis. The equation describing this relationship is **Byerlee's law**, and written as

$$\tau_c = \sigma_N \tan \phi = \mu \sigma_N$$

where  $\phi$  is called the **angle on internal friction** and  $\mu$  is the coefficient of friction. We can create a Mohr construction that illustrates how this equation provides a physical law for predicting when failure will occur in a material:

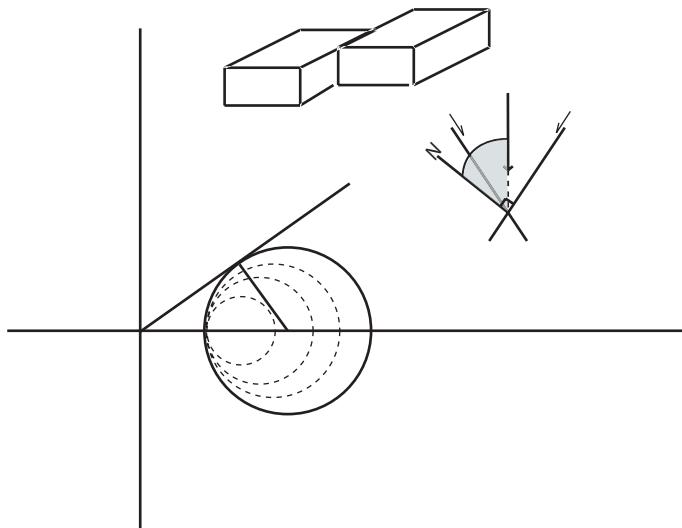


Figure 2: Mohr construction for shear failure according to Byerlee's law (no cohesion). As an exercise, label the figure with  $\tau$ ,  $\sigma_N$ ,  $2\alpha$ ,  $\alpha$ ,  $\sigma_1$ ,  $\sigma_3$ , and  $\phi$ . Also, use this diagram to derive the relationship  $\alpha = 45 + \phi/2$

### 3 Cohesive material

Given the above failure criterion, in the absence of a confining stress, the shear stress for failure will be zero. But materials have strength even in the absence of confining stress – this is known as the **cohesion**. A modified failure criterion incorporating this is the **Mohr-Coulomb** failure criterion:

$$\tau = C + \sigma_N \tan \phi$$

Draw the Mohr circle construction for this.

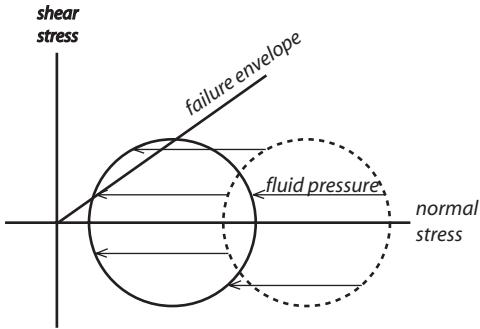


Figure 3:

## 4 Effect of pore fluid pressures

The effect of a pressurized fluid on a fault plane is to decrease the normal stress by that pressure. In this case, the Mohr-Coulomb failure criterion becomes

$$\tau = C + \sigma_{Neff} \tan \phi = C + (\sigma_N - P_f) \tan \phi$$

where  $P_f$  is the fluid pressure. On a Mohr diagram, the effect of fluid pressure is to shift the Mohr circle to the left of the diagram. As fluid pressures go up, the possibility that the Mohr circle intersects the failure envelope is increased.

## 5 Review questions

Lab 1 on stress, particularly the orientation of principal stresses in relation to fault types and tectonic environments. Lab 5 on faults, particularly the problems on joints and Downie's slide.

What does a generalized failure envelope taking into account both tensile and shear failure look like?

In general, what is the effect of higher mean stress (pressure)? What implications does this have for the brittle failure of materials at greater and greater depths?