# Online Markets

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# The VCG Mechanism

Before turning to today's main topic I'll briefly review the VCG mechanism.

A private value social choice problem with transferable utility $(\Theta, p, A, v)$ consists of

1. A set $\Theta_1 \times \Theta_2 \times \cdots \times \Theta_I$ of types for each player.
2. A probability distribution $p$ on $\Theta$.
3. A set $A$ of possible social alternatives
4. Utility functions $u_i : A \times \mathbb{R} \times \Theta_i \to \mathbb{R}$ with $u_i(a, t; \theta_i) = v_i(a, \theta_i) + t$.

The utiltarian solution $a^u$ to $(\Theta, p, A, v)$ is the function $a^u : \Theta \to A$ defined by

$$a^u(\Theta) = \text{argmax}_{a \in A} \sum_{i=1}^{I} v_i(a; \theta_i)$$

A mechanism $m = (S, a, t)$ consists of

1. A set $S = S_1 \times S_2 \times \cdots \times S_I$ of possible strategy profiles.
2. An action function $a : S \to A$.
3. Transfer functions $t_i : S \to \mathbb{R}$.

# The VCG Mechanism

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Given an social choice problem $(\Theta, p, A, v)$ and a mechanism $m = (S, a, t)$ we consider the game where each players observes their types $\theta_i$, choose strategies $s_i \in S_i$, and the mechanism then determines the social alternative and the transfers.

We can think of the second price auction as an example of a mechanism that implements the utilitarian solution as a dominant strategy BNE in the IPV auction environment.

# The VCG Mechanism

An important result due to Vickrey-Clarke-Groves is that can implement utilitarian outcomes much more generally via the VCG mechanism.

Given any player $i$ define $a^u_{-i}$ to be the utilitarian solution ignoring $i$'s preferences, i.e. $a^u_{-i}(\theta) = \text{argmax}_{a \in A} \sum_{j \neq i} v_j(a, \theta_j)$.

The externality that $i$ exerts on others in the social choice problem is

$$\sum_{j \neq i} v_j(a^u(\theta), \theta_j) - \sum_{j \neq i} v_j(a^u_{-i}(\theta), \theta_j).$$

The VCG mechanism $(S_{VCG}, a_{VCG}, t_{VCG})$ is defined by

1. $S_{VCG} = \Theta$.
2. $a_{VCG}(s) = a^u(s)$.
3. $t_{VCG,i} = \sum_{j \neq i} v_j(a^u(\theta), \theta_j) - \sum_{j \neq i} v_j(a^u_{-i}(\theta), \theta_j)$.

Informally, players are asked to directly announce their types, we choose the utilitarian solution given the stated preferences, and players pay for the externalities they impose on others.

The second-price auction is a special case: the winner pays the amount by which they decrease the utility of the bidder who would have won otherwise.

# The VCG Mechanism

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1. $S_{VCG} = \Theta$.
2. $a_{VCG}(s) = a^u(s)$.
3. $t_{VCG,i} = \sum_{j \neq i} v_j(a^u(\theta), \theta_j) - \sum_{j \neq i} v_j(a^u_{-i}(\theta), \theta_j)$.

Theorem: The VCG mechanism implements the utilitarian solution as a truthtelling dominant strategy BNE in any private value social choice problem.

Some things to know about the VCG mechanism are:

1. VCG is very powerful. It applies far beyond the IPV model, with multiple goods, players caring about others' allocations, etc.
2. VCG is the essentially the only way to achieve dominant strategy BNE implementation of $a^u$.
3. VCG is **not** budget balanced. In the second price auction it's important that the high-bidder makes a payment that does not go to the second-highest bidder.
4. The VCG action/announcement spaces can be very large in complex problems. This can make VCG impractical.

Theorem: The VCG mechanism implements the utilitarian solution as a truthtelling dominant strategy BNE in any private value social choice problem.

Proof: We want to show that $s_i = \theta_i$ is a BR to any $\hat{\theta}_{-i}$ announced by the others. Note that

$$\begin{aligned} u_i(\theta_i, \hat{\theta}_{-i}; \theta_i, \theta_{-i}) &= v_i(a^u(\theta_i, \hat{\theta}_{-i}); \theta_i) + \sum_{j \neq i} v_j(a^u(\theta_i, \hat{\theta}_{-i}); \hat{\theta}_j) \\ &\quad - \sum_{j \neq i} v_i(a^u_{-i}(\hat{\theta}_{-i}); \hat{\theta}_j) \\ &= W(a^u(\theta_i, \hat{\theta}_{-i}); \theta_i, \hat{\theta}_{-i}) - h(\hat{\theta}_{-i}) \end{aligned}$$

where $W$ is social welfare and $h$ is a function that does not depend on $\theta_i$.
If player $i$ deviates to $\theta_i'$, the outcome is some possibly different $a'$, and player $i$'s utility is

$$u_i(\theta_i', \hat{\theta}_{-i}; \theta_i, \theta_{-i}) = W(a'; \theta_i, \hat{\theta}_{-i}) - h(\hat{\theta}_{-i})$$

$a^u$ was defined as the maximizer of welfare given the stated types, so the first term is now weakly lower and the second unchanged.
This shows that $s_i = \theta_i$ is a best response to $\hat{\theta}_{-i}$ so truthtelling is a BNE in dominant strategies.

## Google

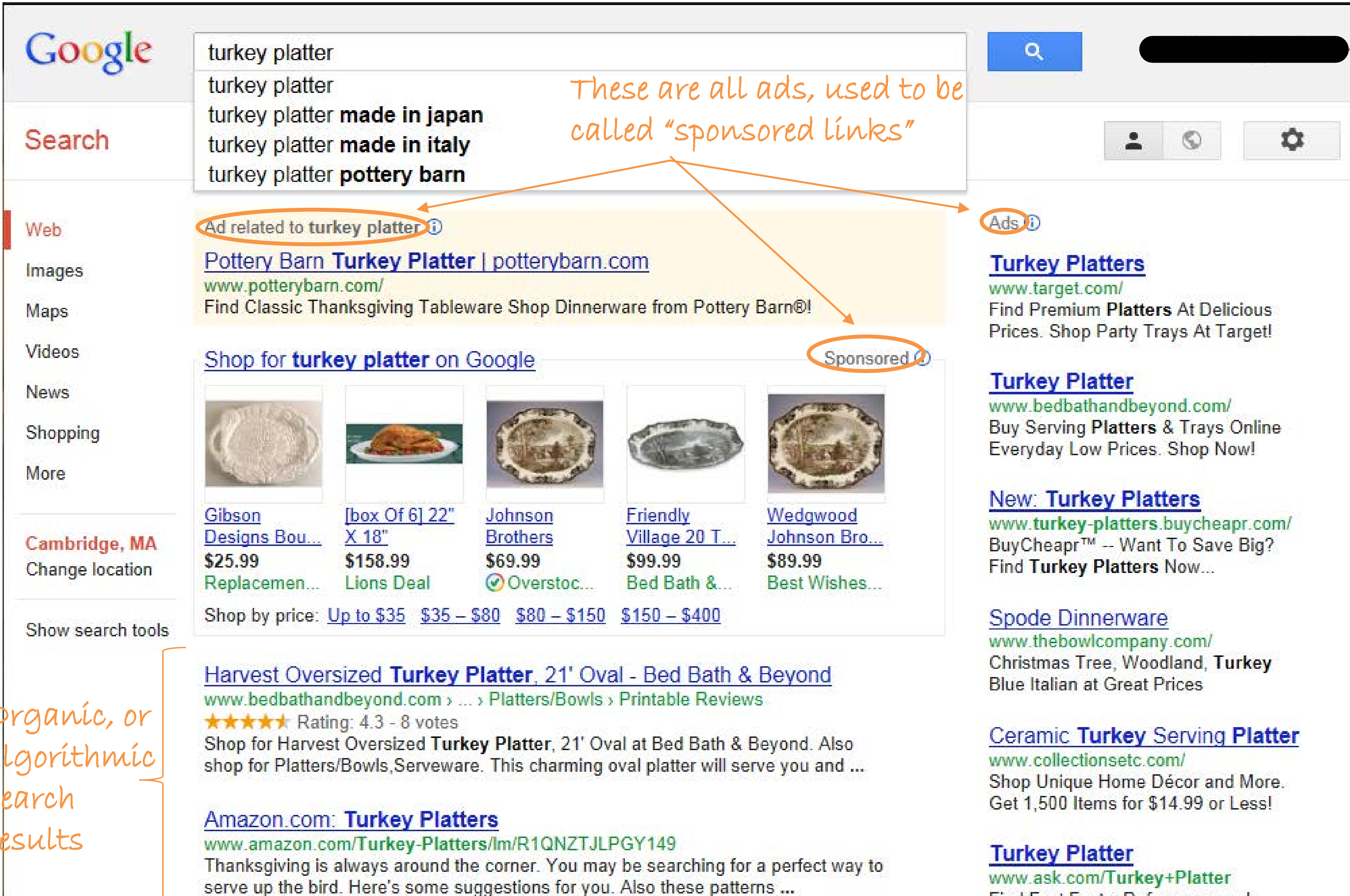
Google passed Seznam (in Czechia) in 2011, Yahoo! (Japan and Taiwan) by 2015, and Naver (South Korea) in 2016. This leaves China and Russia as the only major countries where Google is not #1. Its worldwide market share is reported to be around 92%.

Google's search advertising business is incredibly profitable. It earns high profit margins on about $100 billion in revenue, giving Google the ability to have dramatic effects on many other markets: Android, Gmail, Chrome, Chrome OS (?), Google Docs (?), Google Cloud (?), Google Meets (?), etc.

The US Department of Justice filed suit on October 20, 2020 alleging Google violated the Sherman Act in monopolizing search and search advertising. Several states have sued over ad sales practices and the DOJ may file related claims.

Some basic facts about Google advertising are:

1. Google auctions "sponsored link" ads whenever a search query is entered.
2. The advertising auction produces stable results.
3. Google advertising is highly profitable.
4. Competition from Bing has had little impact on profit margins.




Organic, or algorithmic search results

Not a fixed number of ads, varies with search term

Other search engines, like Bing, do the same thing.

WEB IMAGES VIDEOS MAPS ELECTION SHOPPING MORE

bing

turkey platter

17,300,000 RESULTS

Ads

[Pottery Barn® Tableware | potterybarn.com](www.potterybarn.com)
www.potterybarn.com
Find classic style for every space: tableware, accessories & more

**Turkey Platters** - **Turkey Platters** Online.
www.Target.com
Shop Dinnerware at Target.

Serving **Platters** & Boards | westelm.com
www.westelm.com
Shop Unique Serving **Platters** & Cheese Boards at west elm®.

Spode Woodland **Turkey** - Save 30% On Spode Woodland **Turkey**.
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Ads

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www.marksandspencer.com/**Platters**
Buy Food **Platters** Online at M&S.
Delicious Food to Order From M&S.

Save On Steel **Platters**
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High Quality Restaurant Supplies At Great Prices. Shop Online Today!
webstaurantstore.com is rated ★★★★★ on Bizrate (292 reviews)

**Turkey Platters** for Less
www.Calibex.com/**Turkey-Platters**
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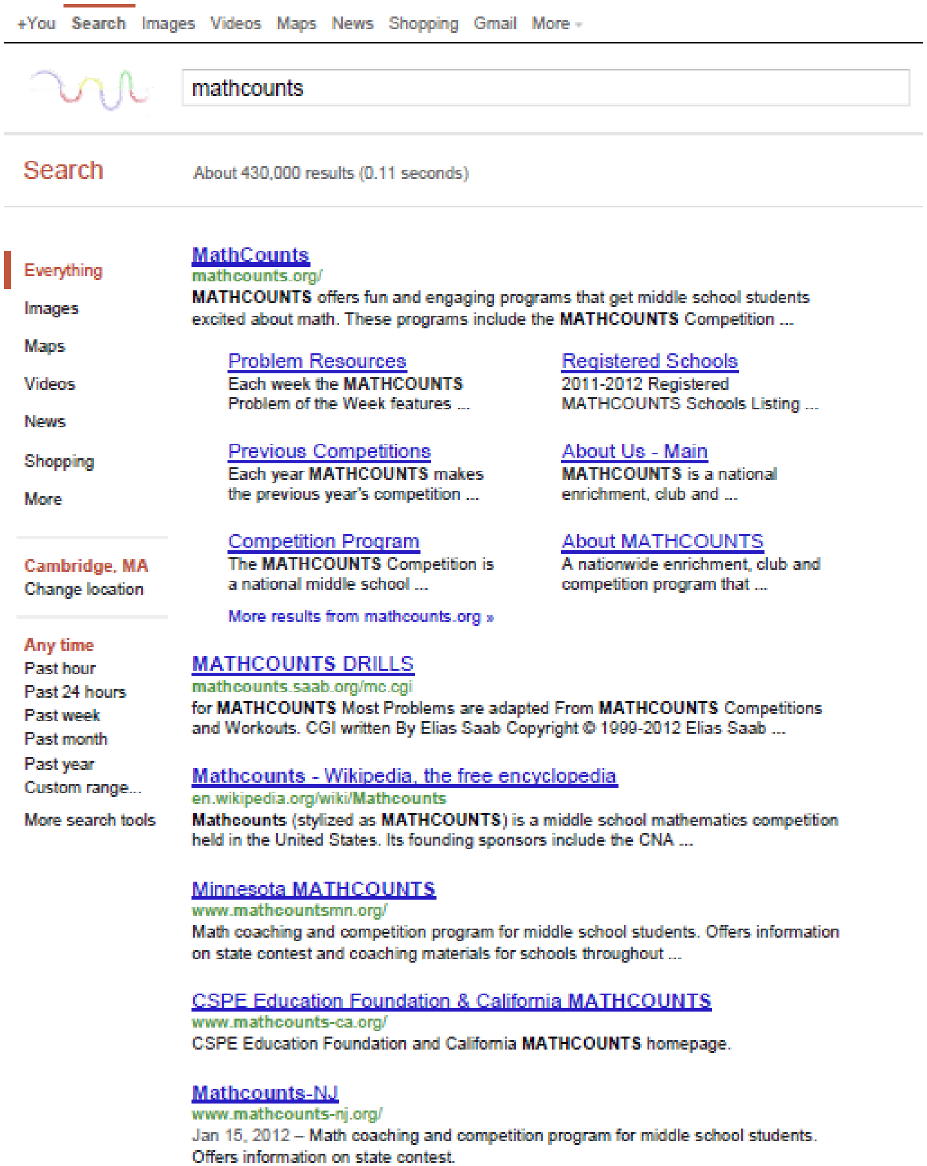
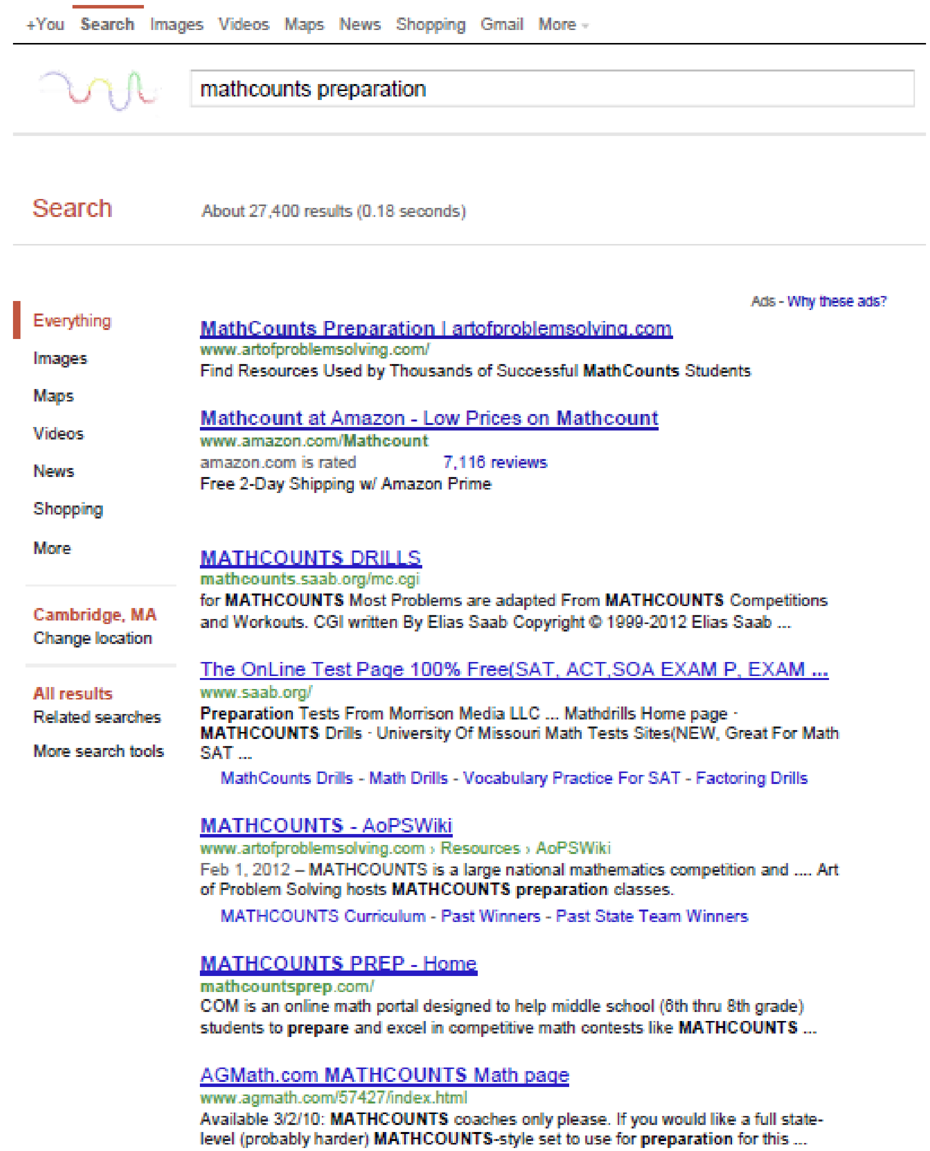
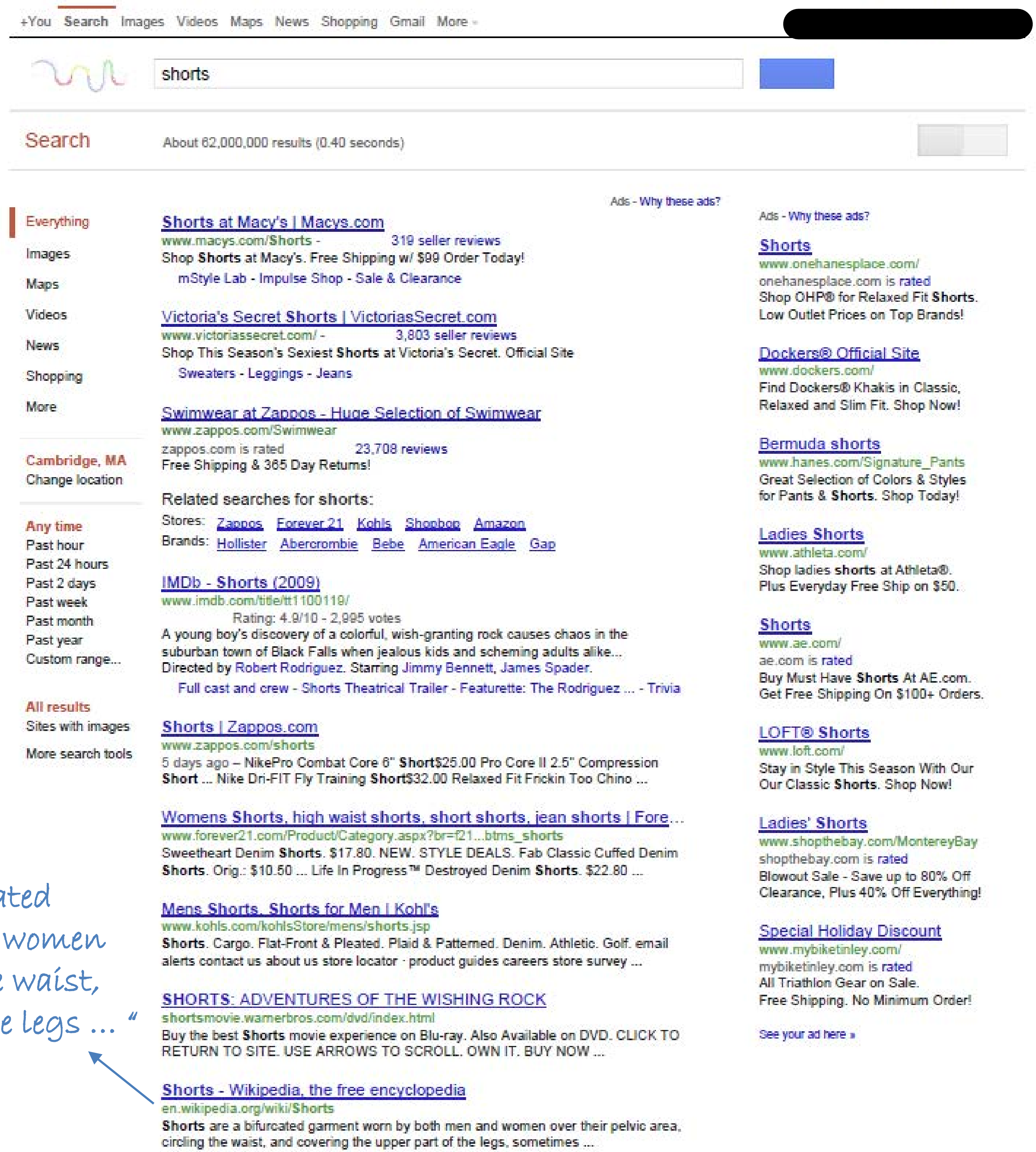
**Personalized Thanksgiving Platters**

# Two examples: Ads vary with commercial potential



And here's an example where no one cares about the organic results really, only the ads:




Wikipedia: "Shorts are a bifurcated garment worn by both men and women over their pelvic area, circling the waist, and covering the upper part of the legs … "

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Welcome, gellison_ge [Sign Out] Help

YAHOO! SEARCH

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Paris Hilton Photos

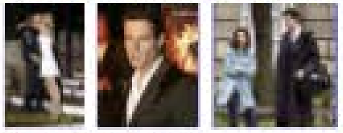
Search Advanced Search

Search Results

1 - 10 of about 76,500,000 for Paris Hilton Photos - 0.21 sec. (About this page)

Also try: paris hilton private parts photos, hot paris hilton photos More...

Paris Hilton - Image Results



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Paris Hilton fan site offers news updates and photo galleries of the heiress and socialite, including magazine spreads and awards show appearances.
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Are You Paris Hilton?
Celebrity quiz: What celebrity are you? Learn which celebrity you are.
QuizRocket.com/what-celebrity-quiz

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Shop paris hilton photos using trusted Epinions reviews.
www.DealTime.com

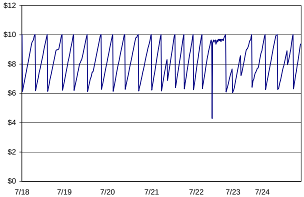
See your message here...

Here's an example from an old Yahoo search page for the term "Paris Hilton." It really exhibits how difficult it can be to provide high quality search results and also be able to fund your operation.

People searching for "Paris Hilton" may want celebrity news, videotapes, or to book a hotel room. They do not want ring tones. And they want to be able to tell before clicking what each link might be getting them.

# Google

1. Google auctions "sponsored link" ads every time a search query is entered.
2. The advertising auction produces very stable results.
   For contrast, the figure below shows the high bid on one keyword on Yahoo! over the course of one week in July 2002




(b) 1 week

3. Google advertising is highly profitable.
   Google may be earning $15-20 billion on 2 trillion searches. Note that 2 trillion searches is less than one search per person per day.
4. Competition from Bing has had little impact on profit margins.

# How Does Google Advertising Work?

Often there are no ads. But on commercially-relevant queries there can be up to three sponsored links on the top of the page and eight more on the right side.

A search engine that wants to maximize its market share will have an incentive to pick ads that consumers want to see. It is hard for a computer program to inspect an ad and determine whether the offer will appeal to consumers.

Advertisers will have information on the likely profitability of their ads. This will be correlated with social welfare if profits and consumer surplus are aligned.

In theory it would be natural to use a VCG mechanism to reveal the values of their ads. But, this is another example where VCG is infeasible. The number of ways to assign up to 11 slots to 50 advertisers is enormous.

# How Does Google Advertising Work?

Google developed a system in which advertising slots are auctioned every time a user enters a query.

- Advertisers submit standing per-click bids for each possible search query, e.g "Paris", "Paris Hotels", "Paris -Hilton"
- When someone types a search query Google identifies applicable bids $b_1, b_2, \ldots, b_N$.
- Bids are multiplied by "quality scores", $w_1, w_2, \ldots, w_N$. The products are ranked from highest to lowest. If $w_1 b_1 > w_2 b_2 > \ldots > w_N b_N$ then bidders $1, 2, \ldots, M$ are chosen as winners for some $M$ with $0 \leq M \leq 11$ and displayed in this order.
- Advertisers pay if their ad is clicked. If ad $k$ is clicked then advertiser $k$ pays Google $\frac{w_{k+1} b_{k+1}}{w_k}$, the lowest that $k$ could have bid and been in the $k$th position.

# Edelman, Ostrovsky, Schwarz (*AER* 2007)

To think about how the mechanism works EOS consider first the following simple model. (For now I'll ignore the weights.)

- Suppose $N$ firms bid for $M$ prizes.
- Prize $k$ consists of getting $z_k$ clicks with $z_1 > z_2 > \ldots > z_M$.
- Assume that bidder $i$ gets payoff $z_k v_i$ if he gets $z_k$ clicks. Assume that $v_i$ is known only to bidder $i$. The others treat $v_i$ as a random variable with known distribution $F_i$.
- Suppose that advertising slots are allocated by the following "clock auction" procedure: The price clock starts at zero. The price rises continuously until all but $M$ bidders drop out. From that point on, whenever a bidder drops out so that only $k-1$ bidders remain, the bidder who just dropped out is awarded position $k$ and assigned a per-click payment equal to the drop-out point of the bidder in position $k+1$.

# Observations

Some initial observations:

1. The GSP auction is not the VCG mechanism.

   The message space is much smaller and in VCG firm $i$ needs to pay

   $$(z_i - z_{i+1})v_{i+1} + (z_{i+1} - z_{i+2})v_{i+2} + \ldots + (z_M - 0)v_{M+1}$$

   rather than $z_i b_{i+1}$.

2. Truthtelling is not necessarily an equilibrium when $M > 1$.

   Example. $N = 3$, $M = 2$, $z_1 = 200$, $z_2 = 199$, $v_1 = 10$, $v_2 = 4$, $v_3 = 2$.

   With truthtelling $\pi_1 = 200(10 - 4) = 1200$

   If 1 deviates to 3 then $\pi_1 = 199(10 - 2) = 1592$.

# Model

The EOS model has an equilibrium in which all bidders who have clinched slots "lie" and bid a little less than a click is worth to them:

**Proposition**

*The clock auction has a unique Bayesian Nash Equilibrium in which strategies are continuous in the types. In this equilibrium:*

1. *All losing bidders stay in until the clock reaches $b_i^*(v_i) = v_i$.*
2. *When $k < M$ bidders remain and the $k+1^{st}$ bidder dropped out at $b^{k+1}$, bidder $i$ will plan to drop out at*

$$b_i^*(v_i; k, b^{k+1}) = v_i - \frac{z_k}{z_{k-1}}(v_i - b^{k+1}).$$

Intuition: Bidders who have clinched a spot on the screen will not bid "truthfully". If you stay in until the clock reaches $v_i$ then you will be very disappointed if someone else drops out at $v_i - \epsilon$ – you'll be left with a profit of just $\epsilon$ per click. It is much better to get fewer clicks at a healthy profit margin by dropping out earlier.

Proof

1. The first claim is immediate. If bidder $i$ drops out before $v_i$ and loses his payoff is zero. Staying in until the clock reaches $v_i$ is better: there is some chance that others will drop out. Staying in past $v_i$ can only produce losses.
2. Payoffs are differentiable so a winner's optimal strategy must be such that he is indifferent to first order between dropping out at $b^*(v_i)$ and $b^*(v_i) + \Delta b$.
   Because dropping out at these two points yields exactly the same payoff if no other bidder drops out in between the two bids, unconditional indifference implies that bidder $i$ must also be indifferent betweend dropping out at the two points *conditional on another bidder dropping out in between* $b^*(v_i)$ *and* $b^*(v_i) + \Delta b$.
   The equation for conditional indifference is:
   $$z_k(v_i - b^{k+1}) = z_{k-1}(v_i - b^*(v_i)).$$
   Solving this equation for $b^*(v_i)$ gives the formula in the Proposition.

# Model

The unremarkable bidding formula has an important Corollary:

**Corollary**

*When players bid as above, the Google auction mechanism results in each player facing the same payment schedule as in the VCG mechanism.*

Sketch of Proof:
Losing bidders pay nothing. This is as in VCG.
The bidder in position $M$ pays $z_M b^{M+1}$. In equilibrium $b^{M+1}$ is the $M+1^{st}$ highest valuation, so this also matches VCG.
The bidder in position $M-1$ pays

$$\begin{aligned} z_{M-1} b^*(v^M; M, b^{M+1}) &= z_{M-1}\left(v^M - \frac{z_M}{z_{M-1}}(v^M - b^{M+1})\right) \\ &= (z_{M-1} - z_M)v^M + z_M v^{M+1}. \end{aligned}$$

Again this is the VCG payment: the first term is what firm $M$ loses by being bumped down from position $M-1$ to position $M$; and the second is what the firm that is bumped off the screen loses.

# Model Implications

1. The fact that Google's mechanism recreates the VCG payments means that the equilibrium is an *ex post* equilibrium. No one wants to change their bid after they see the other bids. This probably explains why the Google mechanism produces stable bids.
2. Another implication of the VCG-equivalence is that the Google mechanism is efficient: the winners are the firms that derive the most value from being listed.

   This suggests why Google may be so popular: if profits and consumer surplus are aligned then the auction may also produce the CS-maximizing page.
3. The VCG mechanism is not budget balanced. In this case, the firms all make nonnegative payments and Google earns the VCG rents.
   From a business perspective this is a stroke for commercial genius: the mechanism that chooses the best links for display results in large payments to Google. No competitor can steal Google's advertisers by offering to let them advertise at a lower price – without the payments you can't select the right advertisers.

## More Issues

The model described above is very nice, but leaves out two other issues that are extremely important in practice.

- In practice Google's most important innovation may have been the "weighted" auction. Each bidder $i$ is in practice assigned a "quality score" $w_i$ and bids are ordered so that $b^1 w^1 > b^2 w^2 > \ldots > b^N w^N$. Payments also reflect the weights: the bidder in position $k$ pays $\frac{b^{k+1} w^{k+1}}{w^k}$ per click.

  Google spends a tremendous amount of money each year researching ways to improve the weights.
- Google also makes liberal use of "reservation prices". If the high bid is less than $r$ then no ad is displayed. Many, many pages have no ads.

  The EOS model does not provide an adequate tool for thinking about welfare implications: in its setup reservation prices are always welfare-reducing because they prevent consumers from seeing ads that are valuable to the advertisers.

# Athey-Ellison, “Position Auctions with Consumer Search,” (*QJE* 2011)

The main idea of our paper is to endogenize the “prizes” in the EOS model as profits obtained from selling to a consumer population. Consumers incur search costs when they click on links and search optimally given beliefs about advertisers.

We have several motivations:

- A more complete model can provide a more complete understanding.
- Thinking about where values come from suggests and motivates directions in which it is natural to change the standard model.
- There is reason to think that answers to auction design questions will change. Reserve prices need not be welfare reducing.
- A complete model lets one talk about consumer and social welfare.

# Model Overview

We build a complete model with rational consumers and profit-maximizing advertisers. The main elements of our model are:

- Consumers have a need. They can meet this need by purchasing from a sponsored-search advertiser.
- Consumers incur search costs whenever they click on a sponsored link.
- Advertisers differ in "quality". Quality is the probability of meeting a consumer's need.
- Quality is private information of the advertiser.
- Consumers gain information about advertiser quality if bids are monotone in quality and screen positions are ordered by bids. This information naturally influences the consumer search process.

# Overview of Results

Some of our observations are:

- Sponsored search auctioneers should be thought of as two-sided platforms that create social surplus by providing information to consumers.
- Some of the standard auction theory results can be generalized to our environment even though the auctions now have “common value” elements.
- Reserve prices have novel effects in our model: reserve prices can improve social welfare by eliminating wasteful search costs and enabling more extensive search; and there is an interesting alignment of consumer and social preferences.
- The standard results on the optimality of click-weighted auctions are not compelling in our model. There are a number of problems: inappropriate selection of firms; loss of information transmission; and incentives for obfuscation.

# Base Model

- Continuum of consumers of unit mass.
  - Visit search page to find sponsored links.
  - Get a payoff of 1 if they meet their need.
  - Consumer $j$ incurs cost $s_j$ from clicking on a link. Assume $s_j \sim G$.
  - Search optimally until need is met or benefit falls below $s_j$.

- N advertisers bid to be sponsored links.
  - Firm $i$ has probability $q_i$ of meeting a random consumer's need.
  - $q_i \sim F[0, 1]$ is private information. Firms' ads convey no information.
  - Get a payoff of 1 if they meet a need.

- Search engine
  - Conducts standard unweighted GSP auction.
  - Displays $M$ sponsored links ordered according to bids.

# Benchmark: Unsorted Lists

A benchmark for comparison is what happens if the advertisements are presented to consumers in a random order.

Define $\bar{q} = E[q_i]$. In that case, the consumer expects each website to meet the need with probability $\bar{q}$.

**Proposition**

*If the ads are sorted randomly, then consumers with $s > \bar{q}$ don't click on any ads. Consumers with $s < \bar{q}$ click on ads until their need is met or they run out of ads. Expected consumer surplus is*

$$E(CS(s)) = \begin{cases} 0 & \text{if } s \geq \bar{q} \\ (\bar{q} - s)\frac{1-(1-\bar{q})^M}{\bar{q}} & \text{if } s < \bar{q} \end{cases}$$

# Consumer Behavior: Sorted Lists

Suppose that the equilibrium of the bidding game is such that the advertisers are ordered on quality. Consumers will form priors based on order statistics from the distribution $F$ and update beliefs downward after every unsuccessful click.

Write $z_i$ for the realization of search $i$.

**Proposition**

*If the firms are sorted by quality in equilibrium, then consumers follow a top-down strategy: they start at the top continue clicking until their need is met or until the expected quality of the next website is below the search cost:*

$$s > \bar{q}_k \equiv E(q^{k:N} | z^1 = \ldots = z^{k-1} = 0)$$

The version of our model with $q$ uniform is a nice special case.

**Lemma**

*If $q \sim U[0,1]$ and sponsored links are ordered on quality, then*

$$E(q^{k:N} | z^1 = \ldots = z^{k-1} = 0) = \frac{N+1-k}{N+k}$$

# Consumer Behavior: Welfare Gains from Sorting Links

In our model sponsored-link lists contribute to welfare by making consumer search more efficient.

When $q$ is uniform and $N$ large consumer surplus is approximately $1 - 2s$ with unsorted links and $1 - s$ with sorted links. These approximations are fairly good even when $N$ is not very large. The figure below shows the curves for $N = 4$.

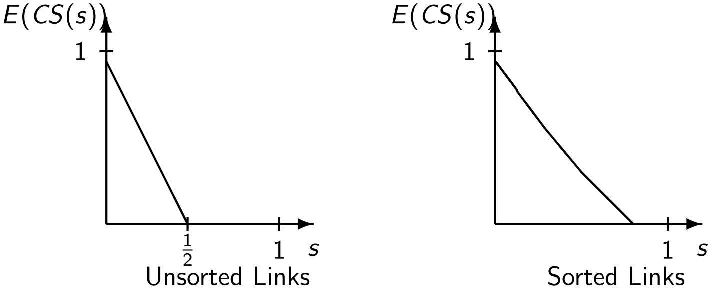



Figure: Consumer surplus with sorted and unsorted links: $N = 4$

## Bidding Game

Consider now the bidding game in which advertisers bid for locations. Suppose $k$ firms remain and the earlier dropout prices were $b^{k+1} \geq b^{k+2} \geq \ldots \geq b^N$.

Each firm $i$ must now decide how long to stay in hoping for a higher slot and when to drop out and accept slot $k$. Equilibrium will be $b^*(k, b^{k+1}; q)$.

The standard EOS model is a private values model. Firms get $(q_i - b^{k+1})z_k$ if they accept slot $k$. If they reach slot $k-1$ instead they get $(q_i - b^k)z_{k-1}$.

Our model has common values. The value to slot $k$ is

$$(q_i - b^{k+1})D_k(q^1, \ldots, q^{k-1}).$$

# Bidding Game

The equilibrium of our model is nonetheless is similar:

### Proposition

*The auction game has a symmetric strictly monotone pure strategy equilibrium. Firms bid up to their value until $M$ firms remain. After this, the dropout point of a firm that has quality $q$ when $k$ bidders remain and the $k+1^{st}$ highest bid is $b^{k+1}$ is given by*

$$b^*(k, b^{k+1}; q) = b^{k+1} + (q - b^{k+1})\left(1 - (1-q)\frac{G(\bar{q}_k)}{G(\bar{q}_{k-1})}\right)$$

<u>Remarks</u>

1. Firms bid up to their true value until they make it onto the list. Then, they start shading their bids. (Note that $G(\bar{q}_k)/G(\bar{q}_{k-1}) < 1$.)
2. When $q$ is small, bids increase slowly with increases in quality because there isn't much gain from outbidding one more bidder.
3. Every time a firm drops out of the final $M$ it is common knowledge that no other firm will drop out for some time.

Proof of Proposition 5

Assume that a symmetric equilibrium exists and that firms play strictly monotone strategies with no bunching of dropout points.

The equilibrium bid $b^*$ will be such that an advertiser is indifferent between dropping out at $b^*$ and at $b^* + db$. Staying in for the extra $db$ makes no difference if no firm drops out, so the firm must be indifferent conditional on another firm dropping out in this interval.

If the firm is the first to drop out its payoff is

$$E\left((1-q^{1:N})(1-q^{2:N})\cdots(1-q^{k-2:N})(1-q)|q^{k-1:N}=q\right)\cdot G(\bar{q}_k)\cdot(q-b^{k+1}).$$

If the firm is the second to drop out in this interval its payoff is

$$E\left((1-q^{1:N})(1-q^{2:N})\cdots(1-q^{k-2:N})|q^{k-1:N}=q\right)\cdot G(\bar{q}_{k-1})\cdot(q-b^*).$$

Indifference gives

$$G(\bar{q}_k)(1-q)(q-b^{k+1}) = G(\bar{q}_{k-1})(q-b^*)$$

Proof of Proposition 5

The indifference condition above is a necessary condition for a PBE with strictly monotone bidding and no simultaneous dropouts. To show that the strategies are in fact an equilibrium we need to do a couple more things:

1. Verify that the strategies do lead to strictly monotone bidding as assumed.
2. Show that the solutions to the indifference equations are optima.
   - If a deviation does not change the order of the listing, then the firm that deviates does not gain.
   - If a firm moves down by dropping out earlier, then the above equations show that dropping out is worse than dropping out immediately after the other firm.
   - The one-stage deviation principle implies that we need only show that a firm does not gain by staying and then dropping out as soon as another firm drops out. This also follows from the above equations.

Suppose search costs are uniformly distributed. An important property is then:

**Proposition**

*Consumer surplus and welfare are maximized for the same reserve price, and given any bidding behavior by advertisers and reserve price policy of the search engine, equilibrium behavior by consumers implies $E(W) = 3E(CS)$.*

Proof

Define GCS = CS + Search Costs and GPS = Advertiser Profit + Search-engine fees.

A search produces one unit of each iff a need is met so $E(GCS) = E(GPS)$.

Welfare is $W = GCS + GPS -$ Search Costs, so we need to show that $E(\text{Search Costs}) = \frac{1}{2}E(GCS)$.

This follows from the optimality of consumer search and the uniform distribution of search costs: each ad is clicked on by all consumers with $s \in [0, E(q)]$, so average search costs are one-half of the expected GCS.

# Reserve Prices

A corollary that is useful for computing socially optimal reserve prices is:

### Corollary

*Suppose that reserve price $r^W$ maximizes social welfare when the search engine has the ability to commit to a reserve price. Then, $r^W$ is an equilibrium choice for a consumer-surplus maximizing search engine regardless of whether the search engine has the ability to commit to a reserve price.*

<u>Proof:</u> Write $CS(q, q')$ for the expected consumer surplus if consumers believe that the search engine displays a sorted list of all advertisers with quality at least $q$, but the search engine actually displays all advertisers with quality at least $q'$.

The optimality of consumer search implies $CS(q, q') \leq CS(q', q')$.

The assumption that advertisers use strictly monotone strategies for any $r$ and that $r^W$ is the socially optimal reserve price imply that $CS(q', q') \leq CS(r^W, r^W)$.

A deviation to a different reserve price yields consumer surplus of $CS(r^W, q')$ for some $q'$. The deviation does not improve consumer surplus because $CS(r^W, q') \leq CS(q', q') \leq CS(r^W, r^W)$.

## Reserve Prices

When consumers have positive search costs it is no longer socially optimal to use a zero reserve price. The calculation is easiest when only one link is displayed.

### Proposition

*Suppose that the list has one position and the distribution of search costs is uniform. Then the optimal $r$ satisfies*

$$r = \frac{1}{2} E(q^{1:N} | q^{1:N} \geq r). \tag{1}$$

Proof:
We can find the social optimum by solving for the Nash equilibrium of the no commitment model with a CS-maximizing search engine. In this model, the search engine must be indifferent as to whether to display any link when it learns that the best firm's quality is $r$. This gives

$$r = \frac{1}{2} E(q^{1:N} | q^{1:N} \geq r).$$

Intuition: Reserve prices improve welfare in two ways: they help consumers avoid wasteful clicks; and thereby enable consumers to click more.

# Reserve Prices

Our alignment theorem has another important implication: there is an inherent conflict of interest between the search engine and advertisers.

**Corollary**

*Advertiser surplus is lower under the profit-maximizing reserve price than under the consumer-optimal reserve price.*

Proof:

The consumer-optimal reserve price maximizes total producer surplus. If the search engine chooses a different reserve price, then it must be that the search engine's share of total producer surplus is larger with this different reserve price. This leaves the advertisers with a smaller share of a smaller pie.

# Weights and Other Extensions

The search engine could obviously do better with more general strategies.

- Reserve prices could vary by position on the screen.
- Displays could be used to convey more information about quality rankings. Current search engines do sometimes leave spaces on the top empty while displaying ads on the side.
- Information about sponsored links could be provided more explicitly.

We discuss a number of other considerations related to weighting bids.

- Clickthrough weights
  - Suppose that each ad has a two dimensional type: A $(\delta, q)$ ad meets each consumer's needs with probability $\delta q$. When consumers read the ad of a $(\delta, q)$ firm, a fraction $1 - \delta$ learn that it cannot meet their needs. The other $\delta$ know that the firm will meet their needs with probability $q$, but still don't know $q$.
  - When $s$ is small using the $\delta$ as CTR weights is approximately optimal. When $s$ is larger it is better to show ads with a higher $q$ and lower $\delta$.
- Relevance weights to combat obfuscation
- Diversity weights

## Anderson and Renault, "Search Direction: Position Externalities and Position Auction Bias"

Anderson and Renault consider a model with endogneous good prices and consumers optimizing search order.

- Consumers have a need that can potentially be met by sellers $i = 1, 2, \ldots, N$.
- Seller $i$ meets the need with probability $1 - \gamma_i$. If seller $i$ meets consumer $j$'s need, it will provide surplus $v_{ij} - p_j$ with $v_{ij} \sim F_i$ on $[\underline{v_i}, \overline{v_i}]$.
- Consumers have cost $s$ per search. Know $\{\gamma_i, F_i\}$ for all firms. Must search firm $i$ to see if need is met and learn $v_{ij}$ and $p_i$. Search optimally given equilibrium prices.

Observations:

1. Under some conditions the model has multiple equilibria, including an equilibrium with every possible search order.

   Equilibrium prices $p_i^*$ are such that consumers buy from the first firm that meets their need. Firms early in the search order are setting prices well below $\underline{v_i}$. This incentivizes the search order and deters further search.

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- S cost $s$. Know $\{\gamma_i, F_i\}$. Search firm $i$ to see if need is met and learn $v_{ij}$ and $p_i$. Search optimally.

Observations:

2. It is tractable to describe the profit-maximizing, wefare maximizing, and consumer surplus maximizing equilibria of this variety.

   Firms are ordered on summary statistics reflecting $\gamma_i$, $\underline{v_i}$, and $\Delta_i$, a measure of the upside potential of learning $v_{ij}$.

   Profit and welfare-maximizing orders place firms with high $\underline{v_i}$ and $\Delta_i$ first. Profit-maxmizing also puts high $\gamma_i$ early, because it's good for the firms if the later firms (which set higher prices) make the sales.

   CS maximizing orders ignore $\underline{v_i}$ (which is fully extracted), and reverse ordering of the other factors.

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- S cost $s$. Know $\{\gamma_i, F_i\}$. Search firm $i$ to see if need is met and learn $v_{ij}$ and $p_i$. Search optimally.

Observations:

3. Position auctions can be modeled as selecting among the equilibria.

   Firms bid for positions, are ordered, and then set the prices that make top-down search in the order determined by the bids an equilibrium.

   The (not fully determined) equilibrium search order depends on the nature of the seller heterogeneity. With differences in "height" ($\underline{v_i}$) high-profit orders result. With differences in "width" ($\gamma_i$) we can get high-welfare orders as in Athey-Ellison.

# Armstrong and Zhou, "Consumer Information and the Limits to Competition"

Armstrong and Zhou discuss retail platform design using an information design approach.

Two firms sell horizontally differentiated products through a retail platform. Consumer $i$ gets utility $v_{ij} - p_j$ if she buys from firm $j$ and utility 0 if she does not purchase. Assume that $v_{ij} \sim F$ on $[\underline{v}, \overline{v}]$ is unknown to the consumer before she visits the platform.

Platform design consists of choosing and committing to a signal structure $\sigma : [\underline{v}, \overline{v}] \times [\underline{v}, \overline{v}] \to \Delta(S)$.

After the platform commits to the signal structure, the firms simultaneously choose prices $p_1$ and $p_2$. Consumers then visit the platform, observe the prices and a signal about their valuations, and purchase from at most one of the firms.

The paper discusses profit-maximizing and consumer-surplus maximizing design.

# First Best Design

The consumer optimal outcome is unattainable. Consumers would like to buy their preferred product at $p = c$. But if consumers are told which product has a higher $v_{ij}$, then the firms are engaged in differentiated product competiton and there will be a positive markup.

The seller-optimal outcome is sometimes possible, but only when there is a very high degree of differentiation. To achieve the seller optimal outcome, each consumer must purchase their most preferred product and firms must price in a way that extracts all surplus. Suppose that each consumer is told which product has the higher $v$ for them. Define $\mu_H \equiv E(v_{i1}|v_{i1} > v_{i2})$ and $\mu_L \equiv E(v_{i1}|v_{i1} < v_{i2})$.

**Proposition**

*If $\mu_H - c > 2(\mu_L - c)$, then $p_1^* = p_2^* = \mu_H$ is an equilibrium of the pricing subgame. These prices extract all consumer surplus.*

# Second Best Design

In othe environments the platform faces an unavoidable tradeoff:

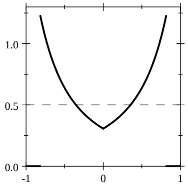
1. Providing consumers with better information about their relative surplus increases gross surplus. This is appealing whether the platform is trying to maximize consumer or producer surplus.
2. A consumer-maximizing platform also wants to minimize equilibrium prices. This requires having many consumers who view firms 1 and 2 as close substitutes.
   A profit-maximizing platforms also wants to maximize equilibrium prices. This requires having few consumers who view firms 1 and 2 as close substitutes.

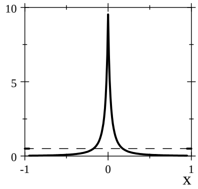
The information-design literature notes that it is often useful to think of signal designs in terms of the distribution over posteriors $g(E(v_1 - v_2|s))$ they induce.

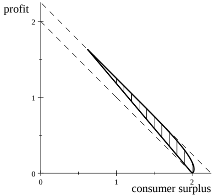
# Second Best Design

The profit-maximizing design (on the left below) uses signals that concentrate beliefs away from indifference.

The consumer-optimal design (in the middle below) creates near-Bertrand competition by giving most consumers very little information.




In a numerical example, there is much more scope to transfer surplus between consumers and producers than to affect aggregate surplus.

*On Monday I'll discuss some empirical papers on online markets probably including*

- *Einav, Levin, Kuchler, Sudaresan*
- *Quan and Williams*
- *Ellison and Ellison*
- *Mayzlin and Chevalier*

*See you then!*