Consumer Search

Glenn Ellison

Diamond Paradox

So far we've focused on differentiation as the driver of markups. Intuitively, consumer search costs should also have an effect. Intuitively, higher search costs could lead consumers to be less informed, increasing equilibrium markups.

This could be particularly important to account for when considering products like credit cards, cell phone plans, and mortgages with little natural differentiation.

Diamond (*JPE* 1971) noted that capturing this intuition requires some subtlety.

- N firms produce homogenous good at cost c.
- Continuum of consumers with identical multiunit demands $D(p)$.
- Assume $(p - c)D(p)$ concave with finite monopoly price p^m.
- Firms simultaneously choose prices $p_1, p_2, \cdots, p_N$.
- Consumers have cost s per price quote. Assume they search optimally getting some number of quotes, then buy $D(p)$ units from the lowest-priced firm they have found.

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One immediate subtlety is the need for a solution concept other than NE. In a NE players act as if they know the other players' strategies. In many search models the whole point is that consumers don't just know the prices.

Diamond used a model of "**sequential search**" with a random permutation: consumers know the equilibrium $p_1^*, p_2^*, \cdots, p_N^*$, but don't know which firm is which, as if the firms have been randomly permuted and placed behind *N* unmarked doors. After getting each price quote they update beliefs and decide whether to search again.

Diamond Paradox

- N firms produce homogenous good at cost c. Choose prices $p_1, p_2, \cdots, p_N$.
- Continuum of consumers. Demands $D(p)$. $(p-c)D(p)$ concave. Monopoly price p^m.
- Consumers have cost s per price quote and do optimal sequential search.

Proposition: If $0 < s < CS(p^m)$, then all PBE in which any consumers search have $p_1^* = p_2^* = \cdots = p_N^* = p^m$.

Proof: The argument that such PBE exist is straightforward. If all firms charge p^m, then all consumers buy from the first firm they visit if it indeed charges p^m. A firm gains nothing by cutting its price, because it still serves just serves just the 1/N consumers that visit it first.

To see there is no other pure strategy equilibrium, suppose we have an equilibrium with $p_1^* \leq p_2^* \leq \cdots \leq p_N^*$ and $p_1^* \neq p^m$. If $p_1^* > p^m$, then firm 1 will benefit from cutting its price to p^m. If $p_1^* < p^m$, then firm 1 will benefit from raising its price to if $p_1^* + \varepsilon$ for some small ε. (The firm still sells to all the same consumers provided $\varepsilon D(p_1^*) < s$, and it makes more per consumer provided $p_1^* + \varepsilon < p^m$.)

We can't have any other firm charges more than p^m because it can cut its price to p^m.

And there also can't be other mixed equilibria – we can make a similar argument about the upper and lower bounds of the supports.

Heterogeneous Search Costs and Price Dispersion

Diamond's model implies that prices jump discontinuously from $p^* = c$ to $p^* = p^m$ if any search costs are present.

A natural class of models in which search costs have a more intuitive continuous effect is models with heterogeneity in search costs. These models also provide an explanation for another real world phenomenon: price dispersion.

- *N* firms produce homogenous good at cost c. Choose prices $p_1, p_2, \cdots, p_N$.
- Continuum of consumers with demands $D(p)$.
- Consumers search optimally. Fraction μ have search cost $s' < 0$ and fraction $1 - \mu$ have costs *s* in some interval $[\underline{s}, \overline{s}]$ with $0 < \underline{s} < \overline{s} < CS(p^m)$.

Proposition (Stahl):

(a) The above model has no pure strategy PBE.

(b) It does have at least one symmetric mixed equilibrium in which firms choose prices from an atomless distribution F with support $[\underline{p}, \overline{p}]$. All such PBE have $\underline{p} > c$.

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Remarks:

1. The mixed equilibrium can be seen as a potential explanation for price dispersion.
2. Prices move continuously with search costs and the the mass μ of "shoppers" as one would expect. Prices converge to c as $\mu \to 1$ or $\overline{s} \to 0$.
3. Equilibria often have U-shaped distributions when *N* is large. Firms sometimes set low prices hoping to sell to all of the shoppers. More often they set fairly high prices to exploit high *s* consumers, but keep some hope they'll sell to everyone.
4. The $N \to \infty$ limit is not what you might expect, but is interesting: F converges to a point mass on p^m.

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<u>Sketch of proof</u>:

The argument for (a) uses a few cases exploiting ε undercutting and ε overcutting.

- Configurations like Diamond's with ties at some $p > c$ so longer work because an ε price cut brings a discrete jump in demand from the "shoppers" with $s' < 0$.
- Configurations with the unequal prices don't work because the lowest-priced firm can raise its price by ε without losing any customers as in the Diamond argument.
- Configurations with ties at $p = c$ also don't work because of ε overcutting. The firm will still sell to consumers who visit it first, and its better to get some profit than no profit.

Heterogeneous Search Costs and Price Dispersion

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<u>Sketch of proof</u>:

For part (b) the main ideas are:

- Existence theorems like Dasgupta-Maskin imply that some equilibrium exists.
- The support $[\underline{p}, \overline{p}]$ of the price distribution can't have $\underline{p} = c$ because firms would be getting zero profits at $\underline{p}$ and can do better by deviating to $c + \varepsilon$.
- The distribution F can't have a mass point at some $\hat{p}$ because firms would then earn more slightly undercutting $\hat{p}$ than at $\hat{p}$. Can also rule out asymmetric equilibria with mass points with slightly more complex arguments: If player 1 has a mass point at $\hat{p}$, then rivals can't price in $(\hat{p}, \hat{p} + \delta]$. But then 1 is better at $\hat{p} + \delta$

Heterogeneous Search Costs and Price Dispersion

Getting closed-form expressions for the equilibrium is hard except in the two-type version with $\underline{s} = \overline{s}$.

In the two-type case (b) said the price distribution has density f on some interval $[\underline{p}, \overline{p}]$.

All consumers with $s > 0$ must in equilibrium buy from the first firm they visit, because otherwise the firm charging $\overline{p}$ will earn zero profits. Hence, equilibrium profits are

$$\pi\left(\underline{p}\right) = \left(\underline{p} - c\right) D(\underline{p})(\mu + \frac{1-\mu}{N})$$

The fact that firms are indifferent over all prices they mix over implies

$$(p - c)D(p)(\mu\big(1 - F(p)\big)^{N-1} + \frac{1-\mu}{N}) = \left(\underline{p} - c\right) D(\underline{p})(\mu + \frac{1-\mu}{N}) \text{ for all } p \in [\underline{p}, \overline{p}].$$

We can solve this equation to find what $F(p)$ must be given any $\underline{p}$.

Note that $\overline{p}$ solves $(\overline{p} - c)D(\overline{p})(\frac{1-\mu}{N}) = \left(\underline{p} - c\right) D(\underline{p})(\mu + \frac{1-\mu}{N})$, so we can find what both $\overline{p}$ and $F(p)$ must be given any (sufficiently small) $\underline{p}$.

Heterogeneous Search Costs and Price Dispersion

The triples $\underline{p}, \overline{p}, F(p)$ that we have found make firms indifferent over all prices in the interval $[\underline{p}, \overline{p}]$. This is not sufficient to show they are NE of the pricing game. For that, we also need to show that firms can't profit from deviating to any other prices.

The potentially problematic deviation is to $\overline{p} + \varepsilon$. This will be profitable if (a) you don't lose any consumers going from $\overline{p}$ to $\overline{p} + \varepsilon$; and (b) you earn a higher per-consumer profit at $\overline{p} + \varepsilon$ than at $\overline{p}$. To ensure that this deviation is unprofitable one must choose $\underline{p}$ so that one of two things hold:

(1) $\overline{p} = p^m$. (In this case (b) fails.)

(2) When a consumer sees price $\overline{p}$ their expected gross benefit $V(\overline{p})$ from searching again is exactly equal to s. (In this case a slight price increase leads all consumers to not buy from the firm and (a) fails.)

To find an equilibrium one would search for values of $\underline{p}$ that satisfy either (1) $\overline{p} = p^m$ or (2) $V(\overline{p}) = s$.

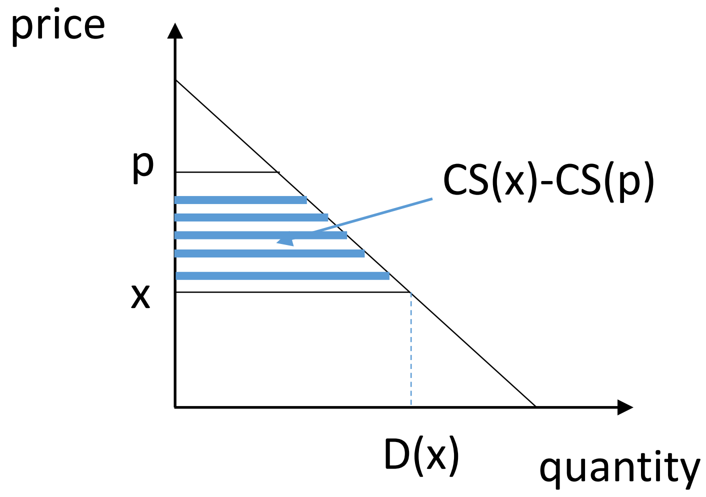
Value of Price Search

Consider price search for a homogeneous good. Suppose that prices are drawn from a distribution with density f on $[\underline{p}, \overline{p}]$ and a consumer has found price p.

The value of one more price quote is $V(p) \equiv \int_{\underline{p}}^{p} (CS(x) - CS(p)) f(x) dx$.

Proposition: The value of search can be written as

$$V(p) \equiv \int_{\underline{p}}^{p} D(x) F(x)\, dx.$$



Proof:

Note that $CS(x) - CS(p) = \int_{x}^{p} D(z) dz$

Changing the order of integration we get

$$\int_{\underline{p}}^{p} (CS(x) - CS(p)) f(x) dx = \int_{x=\underline{p}}^{p} \int_{z=x}^{p} D(z) dz\, f(x) dx$$

$$= \int_{z=\underline{p}}^{p} \int_{x=\underline{p}}^{z} f(x) D(z) dx\, dz = \int_{z=\underline{p}}^{p} F(z) D(z) dz$$

Search with Differentiated Products

The previous models added search costs to Bertrand competition.

We can also add search costs to models with differentiation. Here, search costs can increase markups without creating price dispersion.

- Two firms with constant marginal cost c.
- Consumers with types $(\theta_{i1}, \theta_{i2}) \sim U[0,1] \times [0,1]$ have unit demands with utility

  $v - t\theta_{i1} - p_1$ if they buy from firm 1

  $v - t\theta_{i2} - p_2$ if they buy from firm 2
- Firms simultaneously choose prices p_1, p_2.
- Consumers can pay s to learn p_j, θ_{ij}. Assume $0 < s < t/2$ and consumers use optimal sequential search.

Proposition: For large enough v this pricing game has a symmetric PSNE with

$$p_1^* = p_2^* = c + \frac{1}{2 - \sqrt{2s/t}} t$$

Search with Differentiated Products

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Notes:

1. The Perloff-Salop formula gives $p^* = c + \frac{1}{2}t$ with full information, so this model has price continuous with this for $s \approx 0$ and increasing to $c + t$ as s increases to $t/2$.
2. Search costs do not create price dispersion.
3. For larger s no sales occur. When consumers never get a second quote we have a Diamond-like model where $p^* = p^m \geq v - t$ is the only possible equilibrium. But consumers then have expected CS of at most $t/2$ so they don't get any quotes.

Some intuition: firms considering a price increase trade off the gains on inframarginal consumers against losses from inducing marginal consumers to buy elsewhere. With larger s there are fewer marginal consumers who see both prices (or would at slightly higher p). Hence, the losses on each marginal consumer must be larger. Prices rise until larger per-consumer losses are the offsetting force.

Search with Differentiated Products

Proposition: For large enough v this pricing game has a symmetric PSNE with

$$p_1^* = p_2^* = c + \frac{1}{2-\sqrt{2s/t}}\,t$$

<u>Proof</u>:

A consumer who has found a match quality of $x \equiv 1 - \theta_{i1}$ and a price of p^* at the first firm visited has an expected gain from search of

$$V(x) = t\int_x^1 (u-x)\,du = \frac{1}{2}t(1-x)^2$$

A consumer who sees p_1 will skip the second search and buy $\Leftrightarrow V(x + (p^* - p_1)) \leq s$. In equilibrium $p_1 = p^*$ and this holds $\Leftrightarrow \frac{1}{2}t(1-x)^2 < s \Leftrightarrow x \geq 1 - \sqrt{2s/t} \equiv x^*$.

The NE is the solution to $\frac{\partial \pi_1}{\partial p_1}(p^*, p^*) = 0$. Suppose firm 1 cuts its price to $p^* - dp$ and think about $\pi_1(p^* - dp, p^*) - \pi_1(p^*, p^*)$. Think about gains and losses.

Loss on consumers who would have bought anyway: $\frac{1}{2}dp$.

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In equilibrium $p_1 = p^*$ and this holds $\Leftrightarrow \frac{1}{2}t(1-x)^2 < s \Leftrightarrow x \geq 1 - \sqrt{2s/t} \equiv x^*$.

Loss: $\frac{1}{2}dp$.

visit firm 1 first

continue search

Would have bought from 2 if saw both

Gain: $\left[\frac{1}{2}x^*\frac{1}{t}dp \quad + \quad \frac{1}{2}x^*\frac{1}{t}dp \quad + \quad \frac{1}{2}(1-x^*)\frac{1}{t}dp\right](p^* - c)$

visit firm 2 first

continue search

match with 1 worse, within dp/t

match with 1 worse, within dp/t

visit firm 1 first

Match with 1 in (x*-dp/t,x*) so cut stops search

Equating the FOC is $\frac{1}{2} = \frac{1}{t}(x^* + \frac{1}{2}(1-x^*))(p^* - c) \Leftrightarrow (p^* - c) = t\frac{1}{1+x^*} = t\frac{1}{2-\sqrt{2s/t}}$.

Fixed Sample Search

In some applications, e.g. getting quotes from contractors, time constraints force consumers to choose the number of quotes before seeing any of them.

The model is also popular in structural empirical work because it can be easier to work with than the sequential search model.

- N firms simultaneously choose prices $p_1, p_2, \cdots, p_N$ for a homogeneous good.
- Heterogeneous consumers have $s \sim G$ on $[\underline{s}, \overline{s}]$. Multiunit demands $D(p)$.
- Consumers can pay ks for k quotes.

Suppose equilibrium prices are iid with density f on $[\underline{p}, \overline{p}]$. The value of k quotes is $U_k \equiv \int_{\underline{p}}^{\overline{p}} CS(x) f^{1:k}(x) dx$, where $f^{1:k}(x) = k\big(1 - F(x)\big)^{k-1} f(x)$ is the density of the lowest price with k draws.

Note that $U_1 - U_0 > U_2 - U_1 > \cdots > U_N - U_{N-1}$ because the incremental gains are $U_k - U_{k-1} = \int_{\underline{p}}^{\overline{p}} V(x) f^{1:k-1}(x) dx$ with V increasing and $F^{K-1} \succ F^K$ in the FOSD sense.

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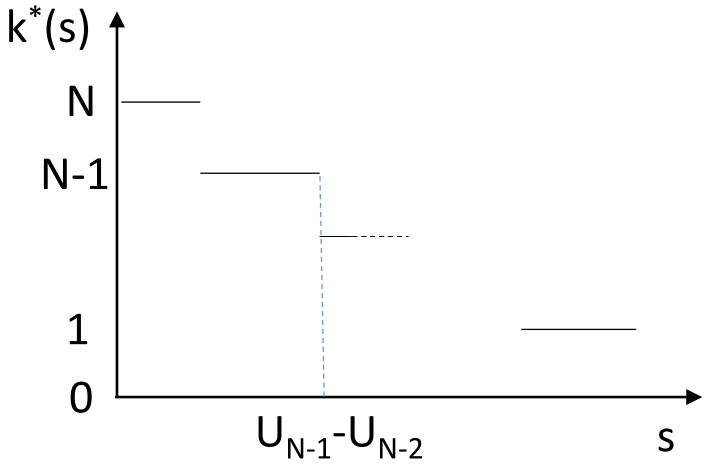


Suppose equilibrium prices are iid with density f on $[\underline{p}, \overline{p}]$.

Incremental gains have $U_1 - U_0 > \cdots > U_N - U_{N-1}$.

Hence, the number of searches that a consumer will choose to conduct is decreasing in s.

With enough heterogeneity in s we can get a dispersed price equilibrium similar to that in Stahl's model with q_k consumers doing k searches.

Such an equilibrium will need to have $q_1 > 1$ and $\overline{p} = p^m$. To find such an equilibrium (numerically), one can solve for F (using firm indifference over prices as in Stahl) and $\underline{p}$ for each guess of $q_1, \cdots, q_N$, and then compute the U_k and $q_k = G(U_k - U_{k-1}) - \overline{G}(U_{k+1} - U_k)$ implied by that F. One searches over possible $q_1, \cdots, q_N$ for a fixed point.

Obfuscation: Search Cost Theories

In the standard models of consumer search it is clear that firms would *collectively* benefit from raising consumer search costs s.

This is not the same as showing that it is individually rational for a firm to raise the cost of learning just its price.

Wilson (*IJIO* 2010) develops one model using a two-stage game:

1. Firms 1 and 2 simultaneously choose search costs s_1, $s_2 \in [\underline{s}, \bar{s}]$.
2. Consumers (and firms) observe s_1, s_2. The firms then choose (unobserved) prices p_1, p_2 and consumers follow an optimal search/purchase policy.

This model turns out not to have an equilibrium with $s_1 = s_2 = \underline{s}$. If firm 1 deviates to a slightly higher search cost, more consumers will visit firm 2 first. Knowing that it has a first-visit advantage firm 2 will want to increase its price. Firm 1 can also raise its price and ends up better off.

The model does have equilibria in which at least one firm chooses $s_j > \underline{s}$.

Obfuscation: Search Cost Theories

Ellison and Wolitzky (*RAND* 2012) develop alternate explanations in models in which consumers do not learn the search cost incurred in getting a price quote until after arriving at a firm.

The explanations rely on a higher search cost making it less likely that consumers will want to conduct a second search. This can arise from:

1. Convex disutility of time spent shopping.
2. Uncertainty over the minimum possible search time. This creates signal-jamming incentives to make consumer think that future searches will be more time consuming than they are.

Multiproduct Search

Zhou (*AER* 2014) considers a model in which consumers want to purchase two differentiated products, each of which can be purchased from either of two retailers.

- Consumers have four-dimensional type $(\theta_{11}, \theta_{12}, \theta_{21}, \theta_{22})$. They get utility $(\theta_{i1} - p_{i1}) + (\theta_{j2} - p_{j2})$ if they buy product 1 from i and product 2 from j.
- If they visit firm i they learn its match qualities and prices: $\theta_{i1}, \theta_{i2}, p_{i1}, p_{i2}$.
- Firms i = 1, 2 simultaneously set prices p_{ik} for products k = 1, 2.
- Consumers visit one firm for free. They then optimally choose whether to pay s to visit the other firm also.

As in the earlier model of differentiated product search we'll look for a PSNE where each firm charges p^* for each product.

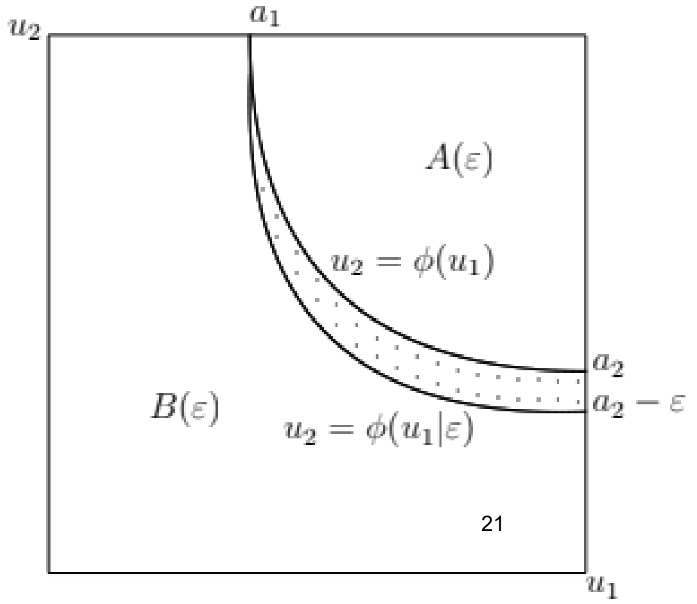
Multiproduct Search

- Consumer utility $(\theta_{i1} - p_{i1}) + (\theta_{j2} - p_{j2})$ if buy 1 from i and 2 from j.
- Firms i = 1, 2 set unobserved prices p_{ik} for products k = 1, 2. Consumers choose whether to pay s to visit a second firm.

$$(\theta_{i1}, \theta_{i2}, p_{i1}, p_{i2}) = \frac{(1 - \theta_{i1} + (p_{i1} - p^*))^2}{2} + \frac{(1 - \theta_{i2} + (p_{i2} - p^*))^2}{2}$$

The set of consumers who buy without a second search is in the upper right, bounded by a circular arc.

- The area is small for s small and big for s large.
- If the firm cuts p_{i2} by ε the arc shifts down and more people don't bother searching.



Multiproduct Search

Some observations from the model are:

1. Relative to the single-product model, the FOCs include an additional "joint-search effect" term that usually* lowers equilibrium prices.
2. The comparative statics of the model need not match those of the single-product model. Prices can *decrease* when search costs increase. (This happens when the θ_{ik} are exponential, but not when they are uniform.)

Intuition for the first is that cutting p_2 also increases demand for good 1.

For the second it helps to note that there are now two effects of raising search costs:

- When s is higher fewer consumers visit both firms. This makes raising prices attractive.
- When s is higher (at least for small s) the arc of consumers on the search boundary is larger. This makes the set of marginal consumers whose search behavior is affected by a dp price cut larger, which makes cutting prices more attractive.

Which effect dominates depends on the distribution of the θ_{ik}.

Next Monday is a holiday. On Wednesday I'll discuss some empirical search papers including

- Sorensen
- Stango and Zinman
- Ellison and Ellison

See you then!

14.271 Industrial Organization I
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