14.271 Midterm Exam
October 26, 2022

Answer <u>all</u> questions. You have 85 minutes in which to complete the exam. Don't spend too much time on any one question. This is a closed-book exam. Do not consult notes, books, or other resources.

1. (25 Minutes – 32 Points) Answer each of the following subquestions CONCISELY. Where we ask questions about papers, please answer the questions asked, and don't just tell us about the papers.

(a) Consider a variant of the traditional competition-on-a-line model in which the disutilities of mismatch are quadratic: a unit mass of consumers get utility $v - t\theta^2 - p_1$ if they purchase firm 1 and utility $v - t(1-\theta)^2 - p_2$ if they purchase from firm 2, with θ uniformly distributed on $[0, 1]$. Assume both firms have a a constant marginal cost c.

What is the demand for firm 1's product as a function of p_1 and p_2 if v is large relative to p_1 and p_2? Without going through all the calculations to compute the equilibrium, how do you think the equilibrium in this model will compare with the equilibrium of the Hotelling model with linear disutilities I went over in class?

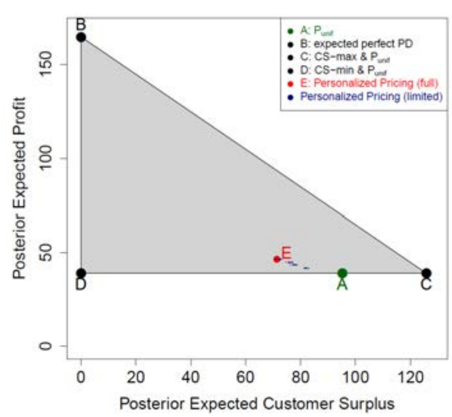
(a sol) Finding the demand requires finding the indifferent consumer. In this case, the consumer is given by

$$
\begin{aligned}
-t(\theta^*)^2 - p_1 &= -t(1-\theta^*)^2 - p_2 \\
\iff \theta^* &= \frac{1}{2} - \frac{p_2 - p_1}{2t}
\end{aligned}
$$

Hence the demand is $D(p_1, p_2) = \frac{1}{2} - \frac{p_2 - p_1}{2t}$.

The demand curve is exactly the same as that of linear disutilities, hence the solution should look the same.

(b) One of the empirical papers in this class included the figure below. What was the paper? Describe very briefly what the paper did to try to estimate the locations of points A, B, C, D, and E? What are the primary economic insights that follow from the location of point E in the triangle? What theoretical paper motivated drawing this triangle? What question did the theory paper pose and what were its main results?



(b sol)The paper is the Dube and Misra paper on personalized pricing. The authors used a randomized expirement to estimate demand and used such demand to estimate consumer and producer surplus based off of different pricing decisions by the firm. This setting was able to estimand demand where the value of the good and the price sensitivity depended on observable attributed.

There are two main implications of the point E, namely that personalized pricing does not get us to the Pareto Fronteir and further it hurts consumers relative to uniform pricing. Further, the profit gain was much smaller relative to the loss in consumer surplus.

It builds off of the theoretical insights of Bergemann, Brooks, and Morris (2015). This paper asked what were the welfare consequences of third degree price discrimination and showed the results are highly dependent on the consumer segmentation you were trying to price discriminate based off of. This paper showed that when plotting CS and Profits on a graph, the range of feasible values makes a triangle. Here, perfect information is at the top and the bottom line goes though standard monopoly pricing point for an uninformed monopolist.

(c) The tables below were taken from two of the papers discussed in class. Which papers were these? What were the primary applied conclusions of the paper on the left? In the table on the left most numbers in each column are very similar. In the table on the right there are more differences within each column. Why do you think this occurs? Should this make us wary of the conclusions reached by the paper that contains the table on the left?

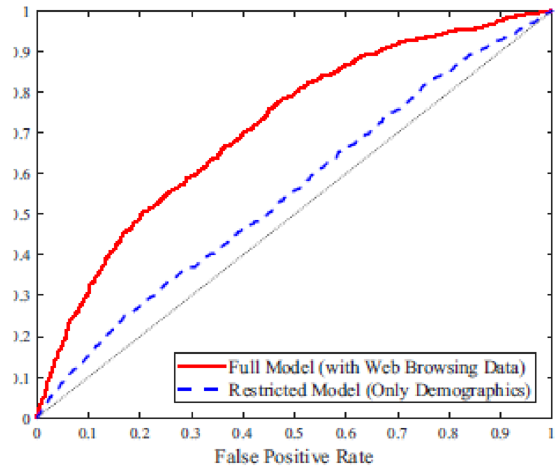
Bud Light	−4.389	0.160	0.019
Budweiser	0.323	−4.272	0.019
Coors	0.316	0.154	−4.371
Coors Light	0.351	0.160	0.019
Corona Extra	0.279	0.147	0.018
Corona Light	0.302	0.151	0.018
Heineken	0.269	0.145	0.018
Heineken Light	0.240	0.112	0.014
Michelob	0.301	0.140	0.015
Michelob Light	0.345	0.159	0.019
Miller Gen. Draft	0.346	0.159	0.019
Miller High Life	0.338	0.159	0.019
Miller Lite	0.344	0.159	0.019

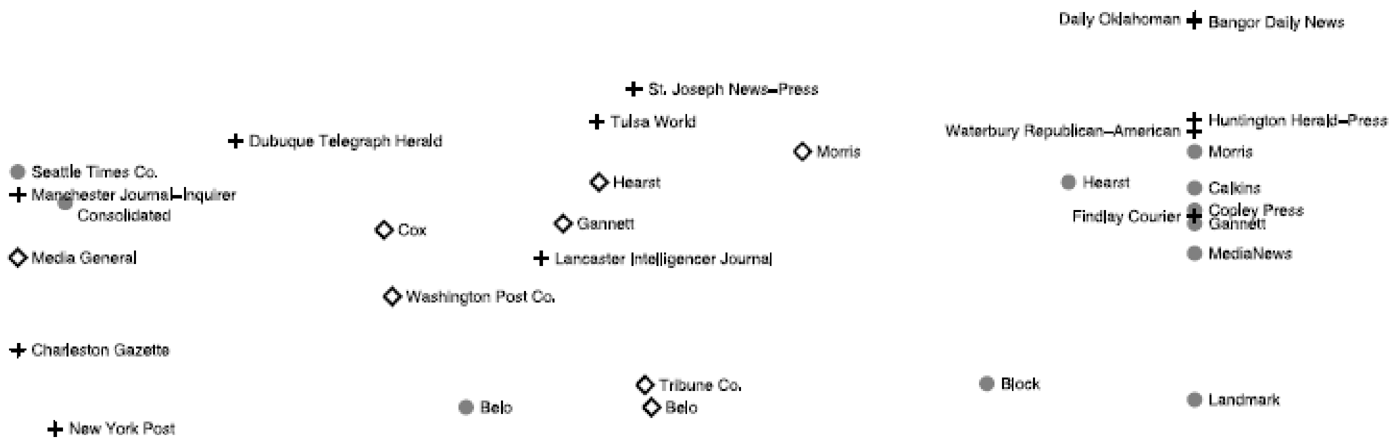
323	−125.933	1.518	8.954
Sentra	0.705	−115.319	8.024
Escort	0.713	1.375	−106.497
Cavalier	0.754	1.414	7.406
Accord	0.120	0.293	1.590
Taurus	0.063	0.144	0.653
Century	0.099	0.228	1.146
Maxima	0.013	0.046	0.236
Legend	0.004	0.014	0.083
TownCar	0.002	0.006	0.029
Seville	0.001	0.005	0.026
LS400	0.001	0.003	0.018
735*i*	0.000	0.002	0.009

(c sol) The two papers are from Miller and Weinberg and BLP. The applied conclusion of the paper on the left was that after a merger, not only was there pricing effects of the merger, it appeared to have enabled collusion between MillerCoors and AB Inbev.

The paper on the right is BLP which estimates demand for cars. It intuitively makes sense that people have different substitution patterns for cars as opposed to beers. I.e. the substitution pattern between a sports car and a mini-van are likely very different than two different light beers. This should not make us wary of the conclusions.

(d) The figures below are from two papers on the syllabus. If you remember or have good guesses for where they came from, discuss in a couple sentences what papers they're from and what purpose they served in the paper.





(d sol) Approximating Purchase Propensities and Reservation Prices from Broad Consumer Tracking by Shiller is the first figure and measures the effects of web-tracking on consumer surplus in the context of Netflix.

The second figure is from Gentzkow Shapiro which estimates how competition impacts newspaper slant. The specific figures plots political contribution to republicans on the X axis and media slant on the Y axis. The plot suggests that slant doesn't appear corelated with donations.

2. (30 Minutes – 36 points)

Consider a variant of Stahl's model of consumer search. Suppose that two firms produce identical goods at zero cost. Suppose that consumers are homogeneous and have unit demands: each consumer is willing to pay up to 12 for one unit of the good. Half of the consumers are "shoppers" who costlessly observe both firms prices. Half are "nonshoppers" who observe the price at one firm and must pay s if they want to learn the price charged by the other firm. Assume that nonshoppers are equally likely to have firm 1 or firm 2 be the firm whose price they observe costlessly. Assume that nonshoppers follow optimal sequential search knowing the equilibrium distribution of prices.

When s is large this model has a symmetric mixed strategy equilibrium in which both firms choose prices from a density $f^*(p)$ with support $[\underline{p}, 12]$ and all consumers purchase.

(a) What is a firm's profit in this equilibrium if it sets $p = 12$?

(a sol) In this equilibrium they will get none of the shoppers and only the shoppers who visit their store first. This corresponds to $\frac{1}{4}$ of the consumers. Thus their profit is $12 \cdot \frac{1}{4} = 3$.

(b) What is the lower bound $\underline{p}$ of the equilibrium price distribution?

(b sol) The lower bound is derived from the indifference condition on the firm. If the firm sets the lowest price on the price distribution they will get all the shoppers and further they will get all the consumers who visit them first. This is $\frac{3}{4}$ of the consumers. Thus $\underline{p}\frac{3}{4} = 3$ hence $\underline{p} = 4$.

(c) What is the equilibrium CDF $F^*(p)$ from which the firms choose prices?

(c sol) The equilibrium distribution is derived from the following indifference condition.

$$3 = \frac{1}{2}(1 - F(p))p + \frac{1}{4}p$$

Here, the left hand side shows what profits the firm must receieve. The first term on the right hand side measures the probability of getting the shoppers(which is the probability the other firm prices higher) and the second term is the profit from the non-shoppers. Solving this expression yields

$$F(p) = \frac{3(p-4)}{2p}.$$

(d) What did I say in class about the typical shape of the equilibrium price distribution in Stahl's model? Sketch the density $f^*(p)$ that corresponds to the CDF found in part (c)? In what way(s) does it differ from what I sketched in class? Without doing any calculations, how do you think you might modify the model to produce an equilibrium more like that I described?

(d sol) One can calculate the PDF by taking a derivative of the CDF above. This calculation yields $f^*(p) = \frac{6}{p^2}$. The pdf is a decreasing function over the range of the interval. Normally

Stahl's model yields to a U-Shaped distribution. However, to get the U-Shaped distribution Stah's model typically considers N, the number of firms, to be large.

(e) The strategy you solved for in part (c) is only an equilibrium if s is at least $\underline{s}$. Give some intuition for why the parameter restriction is needed. Find $\underline{s}$. Describe in a few sentences what you think the equilibirum would look like for $s < \underline{s}$ and how you would go about finding it.

(e sol) The restriction is needed to ensure that the non-shoppers would rather accept the price of 12 as opposed to paying the price s and getting another price quote. For smaller s, the top of the price distribution will be lower than 12. This upper point can be characterized as the indifference point for the consumers between accepting this price and doing one additional search. One can calculate the value of one additional search given that we know the distribution of prices. Here, since the consumer got the maximal price quote from the first firm they visited, they will always accept the second price quote. Thus their expected benefit of search will be

$$\int_4^{12}(12-p)\frac{6}{p^2} = 12 - 6log(3)$$

To charachterize the model for any s, even ones below the critical threshold, we are trying to pin down three things: the maximal price, the minimal price, and the distribution in between. To do so we have a system of equations

$$s = \int_{\underline{p}}^{\bar{p}}(\bar{p}-p)f(p)dp \tag{1}$$

$$\bar{p}\frac{1}{4} = \underline{p}\frac{3}{4} \tag{2}$$

$$\bar{p}\frac{1}{4} = \frac{1}{2}(1-F(p))p + \frac{1}{4}p \tag{3}$$

Where the first expression is derived from the upper price point is the consumer indifference between buying and one more search. The second indifference point is that the firm must be indifferent between getting all the shoppers at the bottom price and none of the shoppers at the top price. The final expression comes from indifference of firms when they must mix.

One did not need to solve the model for full points, but I will do so below.

One can now solve the final expression as we did in part (c) and take a derivative as we did in part (d). This gives us a pdf of $f(p) = \frac{\bar{p}}{2p^2}$. Plugging this back into the consumer indifference expression which is the first equation gives us

$$s = \int_{\underline{p}}^{\bar{p}}(\bar{p}-p)\frac{\bar{p}}{2p^2}dp$$

Further, equation (2) tells us what the lower price is as a function of the upper price. Plugging this in yields

$$s = \int_{\bar{p}/3}^{\bar{p}}(\bar{p}-p)\frac{\bar{p}}{2p^2}dp$$

This is now an expression of only the upper price and s. Solving this equation yields $\bar{p} = s/(1-(1/2)log3)$. Further, we know what the lower price is a function of the upper price and what the distribution must be as a function of the other price. Hence, we have characterized the pricing distribution.

As a check of our solution, one can confirm the distribution converges to a point mass on 0 when $s \to 0$ as both the upper and lower prices converge to this value.

(f) Draw a rough sketch illustrating how you think profits, consumer surplus, and social welfare would vary with s in this model. Do the conclusions one would draw from these graphs seem like good practical insights?

(f sol) In this model, consumers will always purchase and the non shoppers will only ever receive a single price quote. Hence total surplus will remain fixed. Further as the search cost goes down, firm profit can be charachterized by the profit at the top of the price distribution. Since this maximal price goes down with s, then firm profits go down with s. Further, since total surplus is fixed, that implies that consumer surplus must increase. As we saw in (e) past the kink at $\underline{s}$, consumer surplus increases linearly with s. Further, as we saw in (c) the distribution of prices is independent of s above $\underline{s}$.

While the insights themselves may be reasonable: search costs go down yield a weak increase in consumer surplus. The model also yields undesired predictions, such as consumers will only ever get one price quote and that total surplus remains fixed. In reality, one might expect that when search costs go down, consumers are better able to find goods that match their horizontal preferences. This force is absent from the model.

Finally, the consumer surplus as a function of s likely does not have a kink at a critical value of search costs.

3. (30 Minutes – 32 points)

Consider an N firm discrete choice model of demand. Suppose that each consumer i gets utility $v_j - p_j + \epsilon_{ij}$ if they purchase from firm j and utility 0 if they do not purchase. Suppose that the ϵ_{ij} are independent draws from a Normal distribution with mean 0 and variance σ^2.

(a) Suppose first that $N = 2$. Fix values for the v_j and p_j. Draw a sketch illustrating which firm (if any) consumer i will purchase from as a function of $(\epsilon_{i1}, \epsilon_{i2})$. Give an explicit integral or integrals that one could evaluate in order to calculate firm 1's demand as a function of (v_1, v_2, p_1, p_2).

(a sol) Students should draw a graph with a point at $(-(v_1 - p_1), -(v_2 - p_2)$. From this point there will be a horizontal, vertical and 45-degree line. Below the point the consumer will buy neither good, then depending on which side of the 45-degree line we are on the consumer will buy from firm 1 or 2, respectively.

$$\int_{-(v_1-p_1)}^{\infty} \phi(\frac{x}{\sigma})\Phi(\frac{v_1 - p_1 - v_2 + p_2 + x}{\sigma})dx$$

(b) Perloff and Salop provided a result characterizing the equlibrium price in models somewhat like this as a function of N. How did the model that Perloff and Salop analyzed differ from what I have set up? Sketch briefly what they do to make characterizing the equilibrium tractable. Do you remember what the limit as $N \to \infty$ of the equilibrium price of their model would be with normally distributed consumer preferences?

(c) Perloff and Salop have only horizontal differentiation rather than vertical. Under horizontal differentiation, there is a symmetric equilibrium which can be used for tractability. Further, they assume that consumers buy the good that gives them the highest utility and that the outside option gives consumers a utility of ϵ (which follows the same distribution as the preference shocks for the non-outside option products). Here, the equation above in general looks like

$$\int_{-\infty}^{\infty} g(x)\Pi_{j\neq 1}G(x + p_j - p_1)dx$$

This is the demand equation. One can then take a FOC of revenue and embed the symmetric equilibrium solution.

In their model under the continuous distributions prices can diverge to infinity. This can happen if the expected difference in utility from the maximum of n draws vs the maximum of $n+1$ draws is large. Now a firm knows that conditional on getting a sale that consumer must really like their product. However, under the normal distribution, this difference converges to zero due to its thin tails property and hence the price converges to 0.

(c) Tobias discussed the estimation of the canonical random coefficient demand model in which the ϵ_{ij} have an extreme value distribution rather than a normal distribution. Discuss how that model is estimated and how one would need to change the estimation algorithm if one wanted to use normal errors instead.

(c sol) Estimation of these demand models typically requires two coding blocks. An outer loop and an inner loop. The outer loop minimizes our objective function, GMM, over the non-linear parameters in our model. The inner loop, takes a given guess of our non-linear parameters, θ, and for every market solves for the utilities of the products that would rationalize such demands. The logit distribution allows for a closed form solution of predicted demand as a function of qualities. Hence, the inner loop amounts to finding a solution to the equation which equates the predicted quantities and the observed quantities.

With normally distributed errors, we would be unable to find the closed form for the predicted demand as a function of product qualities. Hence, within the inner loop, rather than finding a zero to a known system of equations, one would need to find a zero to a system of equations where the predicted choice probabilities are numerically integrated over all of the epslion's from the normal distribution.

(d) Consider the model above with $N = 1$, i.e. a monopolist selling to consumers with heterogeneous normally distributed valuations for its good and unit demands. Suppose that we want to endogenize the firm's quality v_1 in one of two ways:

(i) The firm might need to incur a fixed cost kv_1^2 to be able to produce a good with quality v_1 at a constant marginal cost of c that does not depend on v_1.

(ii) The firm might be able to choose any v_1 without incurring any fixed cost, but would then have a constant marginal cost of production cv_1^2.

In class I discussed a classic result noting that monopolists will choose product quality optimally in some situations. What does that result imply about whether quality is socially optimal in models (i) and (ii)? If quality is not optimal in either model discuss effects that might make it differ.

(d sol) (i) This isn't the model where Glenn showed optimal quality results. Thinking about it more generally to answer higher or lower, the firm will choose v_1 so that $\frac{\partial \Pi}{\partial v_1} = 0$ so whether it's too high or too low (from a second-best perspective taking monopoly pricing as given) depends on whether dCS/dv1 is positive or negative. Usually this is positive and I assume it would be with normal values because when v is large you're going to want to sell to most people even those with negative epsilons.

(ii) This is a model where Glenn's optimal quality calculation applies. Optimal quality says you choose the optimal v_1 for the marginal consumer (here the consumer with $\epsilon = -(v_1 - p*))$. This is socially optimal here because the marginal consumer like all others has utility that increases one-for-one with product quality so the preferences of the marginal and average consumers coincide.