Problem Set 3

14.271 Fall 2022

This problem set complements some of our discussions in class. The due date is 10/14/22. I recommend that you do the programming exercise in Julia, Matlab, or Python.

1 Deriving logit choice probabilities

Assume that consumer utilities are $u_i = V_j + \epsilon_{ij}$, where ϵ_{ij} is distributed T1EV. We can write the probability that the consumer chooses product j as

$$P_j = \int_{s=-\infty}^{\infty} \left(\prod_{k \neq j} e^{-e^{-\left(s+V_j-V_k\right)}} \right) e^{-s} e^{-e^{-s}} ds,$$

where s is ϵ_{ij}. Derive the closed form logit choice probabilities discussed in class, starting from the above expression.

2 Estimation vie Optimization

2.1 OLS

In this problem you will be estimating OLS by minimizing the sum of squared errors rather than through matrix algebra.

Load airline.txt which contains on-time data for flights within the US. Our regression model is

Delay$_i = \beta_0 + \beta_1$Distance$_i + \beta_2$Departure Delay$_i+$

$$\beta_{3-8}\text{Day of week fixed effects}_i + \varepsilon_i$$

Write a function that calculates the sum of squared errors

$$\sum_i (Y_i - X_i \beta)^2 \tag{1}$$

where $X_i = (\text{Distance}_i, \text{Departure Delay}_i, \text{Day of week fixed effects}_i)$ and $Y_i = $ Delay$_i$ and search over β to minimize the objective function. Compare your results to $\beta_{OLS} = (X'X)^{-1}X'Y$ and report the results.

2.2 Maximum Likelihood

Now we will estimate a logit model using maximum likelihood. Let Y_i be the flight arrival time relative to the schedule. Generate a binary variable for a flight arriving more than 15 minutes late. We will use a logit model to study the probability of a flight arriving late. Let this probability be

$$P(Y_i > 15 | X_i; \beta) = \frac{\exp(\beta X_i)}{1 + \exp(\beta X_i)}$$

where X_i includes a constant, distance, and departure delay.

Write a function to calculate the log likelihood of observing the flight arrival delays in the data:

$$L(\beta) = \sum \ln P(Y_i | X_i; \beta)$$

Search over the parameters β to maximize the log likelihood.

3 Other Approaching to Estimating Demand

Read the paper Cohen et al. (2016) (I uploaded it on Canvas). First, describe what they are doing. Second, provide a critical discussion of their approach. In doing so, try to be precise about potential virtues or shortcomings of their empirical strategy.

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