14.271 Ellison/Salz
Fall 2022

Problem Set 5 Solutions
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Rather than doing a traditional problem set this week, we are asking you to carry out an empirical project using data on the Joint Executive Committee. To do the project you will need a data file which you will be able to get from Stellar. Those of you who have not worked with an econometrics package before or who haven't had a lot of econometrics may do better to work on the project in groups and discuss how to interpret the results you get, but each of you should try to learn how the programs work and think about interpreting the results by yourself. It would be very easy to sit and watch an experienced programmer do all of the empirical tasks in a total of less than fifteen minutes, but it would not be particularly educational. For those of you not familiar with any statistical package, Stata is good one for projects like this one. Some useful commands to look up in the Stata manual are insheet (or import delim), regress, sum, logit, save, and gen.

Part I — Some basics

The file called `porter.csv` is a CSV data file which contains 328 observations on 22 variables. They are in order

WEEK	A variable which takes on values 1 through 328. Week one is the first week of 1880 and week 328 is the sixteenth week of 1886.
QUANTITY	Total JEC shipments of grain (in tons) for the week.
PRICE	The cartels posted price (in cents per 100 pounds) for grain shipments.
LAKES	An indicator variable for whether the Great Lakes were open to navigation.
COLLUSION	An indicator for whether the firms were reported to be colluding.
DM1-DM4	Four dummy variables Porter uses to capture changes in cartel composition.
SEAS1-SEAS13	Dummy variables for each of thirteen four week periods.

(a) Read the data into a statistical package and look at summary statistics to convince yourself that the data was read in correctly. Try a simple OLS regression of log(QUANTITY) on a constant, log(PRICE), LAKES, and (twelve of) the seasonal dummy variables? If you were to view this as an estimate of a demand curve what would the price elasticity of demand be? Why does this number seem unreasonable?

The results of the regressions in (a)–(c) are in Table 1. Price elasticity of demand is 0.64 (I will talk about elasticity in absolute value terms). This is lower than 1. If the firms are jointly acting as a monopolist, the elasticity cannot be lower than 1, according to a monopolist FOC. (Note, however, that it could be lower than 1, if the firms were in a Cournot competition. We are estimating the elasticity of total demand, not individual firm's demand.)

We think that this coefficient is biased due to endogeneity: unobserved by the econometrician, but observed by the firms, positive demand shocks will make the firms price higher, so there is a positive omitted variable bias in the OLS coefficient on price.

[1]Solutions based on those of Adam Harris, Anton Popov, and Sam Grondahl.

	(1)	(2)	(3)	(4)	(5)
VARIABLES	log_quantity	log_quantity	log_quantity	log_price	log_price
log_price	-0.639***	-0.867***	-0.287		
	(0.0733)	(0.134)	(0.911)		
lakes	-0.448***	-0.423***	-0.486**		
	(0.135)	(0.135)	(0.190)		
seas1	-0.133	-0.131	-0.136	0.0414	-0.000965
	(0.0958)	(0.103)	(0.0930)	(0.0660)	(0.0880)
seas2	0.0669	0.0910	0.0297	0.139**	0.0966
	(0.0907)	(0.0949)	(0.132)	(0.0638)	(0.0692)
seas3	0.111	0.136	0.0737	0.191***	0.0888
	(0.0970)	(0.100)	(0.127)	(0.0661)	(0.0651)
seas4	0.155	0.153	0.160	0.0869	-0.0251
	(0.132)	(0.134)	(0.134)	(0.0608)	(0.0734)
seas5	0.110	0.0736	0.165	-0.000254	-0.126*
	(0.128)	(0.130)	(0.197)	(0.0529)	(0.0762)
seas6	0.0468	-0.00606	0.129	-0.0583	-0.154*
	(0.177)	(0.176)	(0.300)	(0.0540)	(0.0826)
seas7	0.123	0.0602	0.219	-0.0997*	-0.179**
	(0.200)	(0.201)	(0.343)	(0.0601)	(0.0777)
seas8	-0.235	-0.294*	-0.144	-0.0685	-0.157**
	(0.175)	(0.175)	(0.325)	(0.0536)	(0.0737)
seas9	0.00356	-0.0584	0.0993	-0.0387	-0.172**
	(0.172)	(0.175)	(0.333)	(0.0558)	(0.0770)
seas10	0.169	0.0858	0.298	-0.133***	-0.267***
	(0.173)	(0.178)	(0.409)	(0.0508)	(0.0794)
seas11	0.215	0.152	0.313	-0.119**	-0.178**
	(0.173)	(0.176)	(0.338)	(0.0573)	(0.0817)
seas12	0.220	0.179	0.283	-0.0207	-0.0800
	(0.170)	(0.171)	(0.264)	(0.0512)	(0.0673)
collusion				0.356***	
				(0.0261)	
dm2					0.132***
					(0.0316)
Constant	9.309***			-1.660***	-1.363***
	(0.116)			(0.0466)	(0.0528)
F-statistic				24.10	7.83
Estimation	OLS	IV (b)	IV(c)	First stage (b)	First stage (c)
Observations	328	328	328	328	328
R-squared	0.313	0.296	0.273	0.488	0.153

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1: OLS and IV demand estimation. Columns (4) and (5) show first stages for columns (2) and (3) respectively.

(b) Try doing the regression instead using instrumental variables with the COLLUSION variable as the instrument for PRICE. How does the reported price elasticity change? Is the estimate closer to that in Porter's paper or that in Ellison's paper and why? How do you interpret the coefficient on the LAKES variable? On the seasonal dummies? What is the R-squared of the regression and what do you make of it?

The elasticity is now 0.87, which may be closer to what we expect. It is closest to Ellison's re-estimation of Porter's equation (0.84). But it is much smaller than the elasticity which Ellison gets, assuming serial correlation in the demand error (1.80).

The coefficient on LAKES means that demand when lakes are open is about 34% lower than when lakes are closed ($e^{-0.423} - 1 \approx -34\%$), holding prices and season constant.

The seasonal dummies indicate differences in demand relative to season 13. It seems that the seasonality in the grain demand is quite high (even though the seasonal dummies are not very significant, they are economically significant).

$R^2 \approx 0.3$ in the IV regression. Remember, however, that we cannot claim that it shows the variation in Y explained by the regressors. This interpretation of R^2 is only valid for OLS, where we have the decomposition $TSS = ESS + SSR$. With coefficients estimated by IV, $\hat{\beta}_{IV}$, the total sum of squares $TSS = \sum_{i=1}^{n}(y_i - \bar{y})^2$ cannot be decomposed into the explained sum of squares $ESS_{IV} = \sum_{i=1}^{n}(X_i'\hat{\beta}_{IV} - \bar{y})^2$, and sum of squared residuals $SSR_{IV} = \sum_{i=1}^{n}(y - X_i'\hat{\beta}_{IV})^2$. (It is not true that $TSS = ESS_{IV} + SSR_{IV}$.)

Stata packages still calculate R^2, using the formula

$$R^2 = \frac{TSS - SSR_{IV}}{TSS}$$

But it does not have a natural interpretation, as in the OLS case. Also, this R^2 may be negative. Intuitively, it is not the purpose of IV estimator to explain variation in y using X.

(c) Try the regression with the DM2 variable instead of COLLUSION as an instrument for price. In what way do the results look "worse" and why do you think this happens.

The results look worse, because demand elasticity (0.29) is even closer to 0 than in OLS. There are two reasons why IV may not work:

- the instrument is not relevant (it is weak)
- the instrument is not valid (it is correlated with error in demand)

Both of these may be at work here. The first stage, reported in column (5) of Table 1 shows that DM2, controlling for seasons, is significant in predicting price at 1% level. However, F-statistic of the first stage is only 7.83, whereas some authors sometimes recommend $F > 10$ as a rule of thumb. So, DM2 may be a weak instrument.

DM2 may also be endogenous. This instrument is equal to 1 only for 15 consecutive observations. Ellison argues in his paper that demand error is better modeled as an AR(1) process. One could imagine the demand errors were higher on average in those periods when DM2 was equal to 1. If that was the case, DM2 would have the same endogeneity problem as the price itself.

(d) Estimate a supply equation as in Porter and Ellison using the LAKES variable as an instrument for quantity. What does the magnitude of the coefficient on COLLUSION tell us about the effect of collusion on prices? What might the coefficient on QUANTITY in this regression indicate about the nature of costs in the JEC?

	(1)
VARIABLES	log_price
log_quantity	0.485***
	(0.111)
collusion	0.430***
	(0.0497)
dm1	-0.192***
	(0.0562)
dm2	-0.199***
	(0.0751)
dm3	-0.346***
	(0.0610)
dm4	-0.0931
	(0.0978)
Observations	328
R-squared	-0.178

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: IV supply estimation. (Instrument for log quantity is LAKES.)

The result of this IV estimation are in Table 2. In periods of collusion firms want to set prices on average 54% higher ($e^{-0.430} - 1 \approx 54\%$) than in periods of price war, holding quantity demanded and DM indicators constant.

Coefficient on log quantity in Porter's framework is equal to $\delta - 1$, where δ is the power of Q in the cost function, so $\delta - 1$ is the power of Q in the marginal cost function. Here the estimated coefficient is positive (0.48) and significantly different from 0, which indicates that JEC has increasing (in Q) marginal costs.

PART II — Model derivation and interpretation

(a) Suppose that rather than the log-log specification of demand you've been using so far, you tried others and found that a linear specification of demand like

$$Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 Lakes_t + u_t$$

seemed most appropriate. Show that for this demand curve the optimal price for a monopolist with a constant marginal cost of c to set is

$$P_t = c - \frac{1}{\alpha_1} Q_t.$$

Given this result, what functional form would you choose for the supply curve in the model?

The monopolist with constant marginal cost solves

$$(P_t - c) Q_t(P_t) \to \max_{P_t}$$

$$Q_t + (P_t - c)Q_t'(P_t) = 0$$

$$P_t = c - \frac{Q_t}{Q_t'(P_t)} = c - \frac{1}{\alpha_1} Q_t \qquad (1)$$

This result suggests that we should estimate a supply curve linear in Q.

(b) What pricing rule would result with this demand curve if the industry instead consisted of perfectly competitive firms with total costs of the form $c(Q_t) = c_0 Q_t + c_1 Q_t^2$ setting price equal to marginal cost? (For extra credit comment on whether this is the equilibrium of a Bertrand-like pricing game). Could one use an approach like Porter's to distinguish between these two models of behavior? Talk about why this is an important question.

In this setting we would have

$$P_t = c_0 + 2c_1 Q_t \qquad (2)$$

Note that this equation is also linear in Q, just like equation in part (a). Coefficients in both equations are positive (note that $\alpha_1 < 0$).

Suppose we estimated demand parameter $\hat{\alpha}_1$, and estimated a supply parameter which was equal to $-\frac{1}{\hat{\alpha}_1}$. Does this mean that we would choose between two models in (a) and (b) in favor of the first one? No, because it could easily be that $2c_1 = -\frac{1}{\alpha_1}$.

So, the two models cannot be distinguished.

How can we think of Porter's approach in light of the model in part (a)? Porter specifies two regimes, in which the conduct parameter is different. Adding conduct parameter θ to equation 1 would give a supply equation of the form

$$P_t = c - \frac{\theta}{\alpha_1} Q_t$$

So, using Porter's approach, we would need to estimate two slopes of Q, corresponding to the two regimes: $-\frac{\theta_{\text{collusion}}}{\alpha_1}$ and $-\frac{\theta_{\text{price war}}}{\alpha_1}$

However, the model in (b) could also generate two different slopes of Q, if the parameter c_1 was switching in time from one value to another.

So, we would not be able to utilize Porter's approach to distinguish between two models of behavior. We do need to make some identification assumptions to have a distinction between conduct and marginal cost slope.

For example, if we wanted to estimate a conduct parameter in a model which also allowed costs to be linear in Q, we would need to assume: 1) marginal cost function remained constant in time; 2) in a price war regime $\theta_{\text{price war}} = 0$ (then we would be able to identify the slope of the marginal cost from the demand variation in the price war regime).

This is an identification question, which seems to be quite important. If we conclude that the industry is behaving collusively, when in fact it is perfectly competitive with increasing marginal costs, that could potentially lead to applying a wrong regulatory policy to the industry.

Extra credit question: Yes, pricing equation

$$P = c_0 + 2c_1 Q$$

could be an equilibrium of a Bertrand pricing game.

To see this, assume an industry of N identical firms, each producing $q = \frac{Q}{N}$. Derive the individual cost functions of each of the N firms:

$$c_i(q) = \frac{1}{N}c(Q) = \frac{1}{N}\left(c_0 Q + c_1 Q^2\right) = c_0 q + c_1 N q^2$$

In the proposed equilibrium each firm earns profits

$$\pi_i(q) = Pq - c_i(q) = c_0 q + 2c_1 N q^2 - c_0 q - c_1 N q^2 = c_1 N q^2 > 0$$

Consider possible deviations for firm i. Firm i will not want to deviate to a price higher than P, because then it gets 0 demand, and its profits fall to 0.

If firm i undercuts the equilibrium price, it will get the entire market, and it will need to pay enormous costs to supply the quantity demanded:

$$\pi_i^d(q) = PQ - c_i(Q) = c_0 Q + 2c_1 Q^2 - c_0 Q - c_1 N Q^2 = c_1(2-N)Q^2 \leq 0$$

Deviations to lower prices will be even worse due to convexity of the individual costs.

(c) Suppose that demand is linear, but that the opening of the Great Lakes also affects the slope of demand and that there are additive seasonal shifts in demand so that the correct specification of demand is

$$Q_t = \alpha_0 + \alpha_1 P_t + \alpha_2 Lakes_t + \alpha_{3-14} Seasxx_t + \alpha_{15} Lakes_t P_t + U_{1t}.$$

How would a monopolist with constant marginal costs set prices in such an environment?

The derivation is the same as in part (a), but now $Q_t'(P_t)$ is different:

$$P_t = c - \frac{Q_t}{Q_t'(P_t)} = c - \frac{1}{\alpha_1 + \alpha_{15} Lakes_t} Q_t$$

Note that Lakes is a dummy variable, so we may rewrite this equation as

$$P_t = c - \frac{1}{\alpha_1} Q_t + \left(\frac{1}{\alpha_1} - \frac{1}{\alpha_1 + \alpha_{15}}\right) Q_t Lakes_t$$

We may also introduce conduct parameters θ_w for the price war periods and θ_c for collusion periods (remember that they apply to the entire $-\frac{Q_t}{Q_t'(P_t)}$ part of the supply equation):

$$P_t = c + [\theta_w + (\theta_c - \theta_w) Collusion_t] \left[-\frac{1}{\alpha_1} Q_t + \left(\frac{1}{\alpha_1} - \frac{1}{\alpha_1 + \alpha_{15}}\right) Q_t Lakes_t\right] \tag{3}$$

This suggests the following estimation equation for supply:

$$P_t = \beta_0 + \beta_1 Q_t + \beta_2 Q_t Collusion_t + \beta_3 Q_t Lakes_t + \beta_4 Q_t Lakes_t Collusion_t + U_{2t} \tag{4}$$

(d) Estimate the demand equation above and a supply equation motivated by the behavior of a monopolist with constant marginal costs using instrumental variables. (Try the Collusion variable and the Collusion variable interacted with the Lakes variable as the two instruments.)

I estimate demand and supply equations as specified in part (c). I use Collusion and Collusion*Lakes as instruments (for P and P*Lakes) in the demand equation. I use Lakes, Lakes*Collusion, and all of the Season variables and their interactions with Collusion as instruments (for Q, Q*Collusion, Q*Lakes, and Q*Lakes*Collusion) in the supply equation. This assumes that in the supply equation endogeneity only comes from Q, and not from Collusion, or Lakes. The results are in Table 3.

What price elasticity of demand is indicated by the estimates (evaluate the elasticity when price and quantity are at their sample means)?

Price elasticity at the average quantity and price when lakes are closed is

$$\eta_0 = -89805 \cdot \frac{\bar{P}_0}{\bar{Q}_0} = -0.82$$

When lakes are open,

$$\eta_1 = (-89805 - 5306) \cdot \frac{\bar{P}_1}{\bar{Q}_1} = -0.98$$

I used mean prices and quantities when lakes are closed and open, respectively, to estimate elasticities. Elasticity at the average quantity and price is higher when lakes are open. This makes sense, because there is a possibility to substitute to water shipping.

What do the results imply about the percent difference between the equilibrium prices in the collusive and non-collusive regimes (assuming that conditions are such that all of the other dummy variables in the equations are set to zero)?

If all other dummies are 0, we have demand

$$Q_t = \alpha_0 + \alpha_1 P_t$$

and supply

$$P_t = \beta_0 + \beta_1 Q_t + \beta_2 Q_t Collusion_t$$

Plugging in Q_t and solving for P_t, we have two price levels in equilibrium. In price war

$$P_w = \frac{\beta_0 + \beta_1 \alpha_0}{1 - \alpha_1 \beta_1} \approx 0.235$$

In collusion

$$P_c = \frac{\beta_0 + (\beta_1 + \beta_2)\alpha_0}{1 - \alpha_1(\beta_1 + \beta_2)} \approx 0.291$$

In equilibrium, prices are on average 24% higher during collusion than during price war.

If you assume that price wars involved marginal cost pricing what would you conclude about the degree θ of collusion in the collusive periods?

Marginal cost pricing in price wars means that $\theta_w = 0$ (note this is rejected by our estimated regression: if we do F-test of coefficients on Q and $Q * Lakes$ both equal to 0, $F = 23.76$). Then the conduct parameter may be estimated from the coefficient on $Q * Collusion$:

VARIABLES	(1) quantity	(2) price
price	-89,805**	
	(35,103)	
lakes_price	-5,306	
	(37,989)	
lakes	-11,703	
	(9,434)	
seas1	-2,008	
	(2,556)	
seas2	3,259	
	(2,611)	
seas3	3,878	
	(2,544)	
seas4	8,622**	
	(3,941)	
seas5	6,569*	
	(3,841)	
seas6	4,478	
	(4,558)	
seas7	6,958	
	(4,999)	
seas8	-1,761	
	(4,477)	
seas9	1,992	
	(4,553)	
seas10	5,474	
	(4,623)	
seas11	6,756	
	(4,620)	
seas12	6,530	
	(4,608)	
quantity		-9.66e-07**
		(4.67e-07)
q_collusion		2.07e-06***
		(3.89e-07)
q_lakes		-2.30e-06***
		(3.47e-07)
q_lakes_collusion		1.11e-06***
		(4.27e-07)
Constant	51,041***	0.264***
	(9,592)	(0.0149)
Observations	328	328
R-squared	0.306	0.442

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: Demand and supply estimation with linear functions.

$$2.07 * 10^{-6} = \hat{\beta}_2 = -\frac{\hat{\theta}_c}{\hat{\alpha}_1} = \frac{-\hat{\theta}_c}{-89805}$$

$$\hat{\theta}_c \approx 0.186$$

which roughly corresponds to Cournot conduct with 5 firms. If taken seriously, this estimation suggests that the firms are only colluding up to Cournot level of output, and not even close to monopoly.

Calculate an alternate estimate of θ by focusing on another term in the supply equation. How do the estimates compare?

Another estimate of θ_c comes from the coefficient on β_4:

$$1.11 * 10^{-6} = \hat{\beta}_4 = \hat{\theta}_c \left(\frac{1}{\hat{\alpha}_1} - \frac{1}{\hat{\alpha}_1 + \hat{\alpha}_{15}} \right) = \hat{\theta}_c \left(\frac{1}{-89805} - \frac{1}{-95111} \right)$$

$$\hat{\theta}_c \approx -1.79$$

which makes no sense. Of course, we may have multiple (non-exclusive) problems here. We already said that assumption $\theta_w = 0$ is not consistent with the estimated regression. Our instruments may be weak or not valid. Our model may be misspecified.

Part III — More on the theory

Consider a repeated duopoly model. Firms 1 and 2 choose quantities q_{it} at $t = 0, 1, 2, \ldots$. As in the Cournot model, the firms' products are undifferentiated and a market clearing condition determines the market price. Assume that demand fluctuates over time, so that this market price is $P_t(q_{1t}, q_{2t}) = A_t - (q_{1t} + q_{2t})$. Suppose that the A_t are independent random variables with $A_t \sim U[0, 12]$. The firms do not know A_t when they choose q_{it}. Firm i cannot see q_{-it} and therefore cannot be sure what A_t was even after seeing P_t. Assume that firms have no costs of production.

(a) Find the fully collusive output q^m and the Cournot equilibrium of the one period game. What are the per firm profits in each? What would the distribution of market prices be in each?

First note that part (c) of this problem specifically asks about $P < -3$ observed, so it is fine to assume that prices may be negative in this problem.

Under full collusion the firms jointly solve

$$\max_Q \mathbb{E}\left[(A - Q) \cdot Q\right]$$

giving the FOC

$$\mathbb{E}[A - 2Q] = 0 \implies q^m = 3 \implies q_i^m = \frac{3}{2} \qquad (\text{FOC}_Q)$$

The distribution of prices will be $\sim \mathcal{U}[-3, 9]$, hence expected per-firm profits will be

$$\mathbb{E}[\pi_i^m] = \mathbb{E}[p q_i^m] = q_i^m \mathbb{E}[p] = \frac{3}{2} \cdot 3 = \frac{9}{2}$$

Under Cournot competition each firm solves

$$\max_{q_i} \ \mathbb{E}[(A - q_i - q_{-i}) \cdot q_i]$$

giving the FOC

$$\mathbb{E}[A - q_{-i} - 2q_i] = 0 \implies q_i^c = 3 - \frac{1}{2}q_{-i}^c \tag{FOC$_{q_i}$}$$

Solving for symmetric best responses in turn yields

$$q_i^c = 2 \implies Q^c = 4$$

We can similarly determine that prices are $\sim \mathcal{U}[-4, 8]$ and expected per-firm profits will be $\mathbb{E}[\pi_i^c] = 4$.

(b) Suppose that A_t is observed after the firms choose q_{it} but before they choose q_{it+1}. For what discount factors could the firms sustain collusion by choosing $q_{it} = q^m/2$ as long as no deviation has been observed and permanently reverting to the Cournot equilibrium if any firm has ever deviated?

The optimal one-period deviation solves

$$\begin{aligned} q^d &= \arg\max_q \quad q \cdot (\mathbb{E}[A] - q_i^m/2 - q) \\ 2q^d &= 6 - \frac{3}{2} \\ q^d &= \frac{9}{4} \end{aligned}$$

yielding profit

$$\pi^d = \frac{9}{4} \cdot \left(6 - \frac{3}{2} - \frac{9}{4}\right) = \frac{81}{16}$$

Hence the incentive constraint to sustain collusion is

$$\begin{aligned} \pi^m(1-\delta)^{-1} &\geq \pi^d + \delta\pi^c(1-\delta)^{-1} \\ \frac{9}{2} &\geq \frac{81}{16}(1-\delta) + 4\delta \\ \delta\left(\frac{81}{16} - \frac{64}{16}\right) &\geq \frac{81}{16} - \frac{72}{16} \\ \delta &\geq \frac{9}{17} \end{aligned}$$

which establishes the bound on δ required to sustain collusion under the specified set of strategies.

The "punishment" phase is the Nash equilibrium of the static game, so after someone has deviated punishing is SPNE, and the entire equilibrium is thus SPNE.

(c) Now go back to the original assumption that A_t is never observable. Suppose the firms try to sustain collusion via strategies that are initially fully collusive and permanently revert to the Cournot equilibrium if the firms ever observe $P_t < -3$. For what discount factors will this punishment make it unprofitable for the firms to deviate to $q^m/2 + dq$? Discuss the additional conditions that would have to be satisfied for this profile to be an equilibrium, and how you could construct collusive equilibria for the set of discount factors you've identified.

As established in (a), under the perfectly collusive equilibrium aggregate $q^m = 3$ so the price distribution is $P \sim \mathcal{U}[-3, 12]$. Increasing one's own quantity by dq shifts the distribution to $\mathcal{U}[-3 - dq, 12 - dq]$. The "getting caught" set is the interval $c = [-3 - dq, -3)$, and the associated probability measure is $\mu = \frac{dq}{12}$. A deviation to $q^m/2 + dq$ yields flow profit

$$(\mathbb{E}[A] - 3 - dq)\left(\frac{3}{2} + dq\right) = (3 - dq)\left(\frac{3}{2} + dq\right)$$

Incentive compatibility thus requires

$$\frac{9}{2}(1-\delta)^{-1} \geq (3/2 + dq)(3 - dq) + \delta\left[\frac{dq}{12} \cdot 4(1-\delta)^{-1} + \left(1 - \frac{dq}{12}\right) \cdot \frac{9}{2}(1-\delta)^{-1}\right]$$

$$9 \geq (3 + 2dq)(3 - dq)(1 - \delta) + \delta\left[9 - \frac{dq}{12}\right]$$

$$\delta\left[(3 + 2dq)(3 - dq) - \left(9 - \frac{dq}{12}\right)\right] \geq (3 + 2dq)(3 - dq) - 9$$

$$(37 - 24dq)\delta \geq (36 - 24dq)$$

Define $\underline{\delta}(dq) := \frac{36 - 24dq}{37 - 24dq}$. For $0 < dq < \frac{24}{36}$, $\underline{\delta}(dq) > 0$, and we can sustain collusion with $\delta > \frac{36-24dq}{37-24dq}$.

In the limit, when $dq \to 0$, $\underline{\delta} \to \frac{36}{37}$, that is, to sustain collusion when very small deviations dq are permitted (under this detection rule), we need a δ close to 1 (a consequence of the low probability of detection).

For $dq \geq \frac{24}{36}$ the inequality above is true for any $\delta \in [0, 1]$ (that is, we can sustain collusion for any $\delta \in [0, 1]$).

Overall, the more we can bound deviation dq away from zero, the wider range of discount factors will support collusion.

One natural question is why in the first inequality above do we use the cooperative/collusive value in the "not getting caught" state of the world – this is the last term on the right. One might think that we should instead use some other maximized value function: maybe there is some deviation that I can play forever that will increase this continuation value which, recomputing the condition above, will tend to reduce the denominator (tightening the condition). This intuition happens to be incorrect based upon the **one-shot deviation principle**, which is due originally to Blackwell (1965).

(d) How is the equilibrium described above similar to and different from the equilibrium that motivates Porter's empirical work and the equilibrium of the two-state version of the Green-Porter model described in Tirole's text (and in class)? Do you think the equilibrium would be a good one to use to motivate tests for collusion?

The general idea of the two games is similar: demand equation is not observed perfectly, so firms need to infer whether a competitor deviated from the market outcome (price). However, in the game analyzed here there are two significant differences:

- Unlike Green-Porter, on the equilibrium path no punishment phase will ever start. This is because nobody deviates in a collusive equilibrium, and so the price never falls below -3.
- If someone did deviate, the punishment phase in this game would never end. Whereas in Green-Porter the punishment phase lasts for a finite number of periods, and then firms revert back to collusion.

Because of these two features this model would not be a good one to motivate tests for collusion. (If we take the model literally, there will never be a switch from collusion to a price war.)

Code:

```
/*

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Problem 2, PS4, 14.271 2018

*/

* workdir
global workdir "//bbking2/popov/winprofile/mydocs/Documents/14.271/PS4/"

* load data
import delimited "${workdir}porter.csv", clear

* generate variables
gen log_quantity = log(quantity)
gen log_price = log(price)
gen constant = 1

* parts a-c
reghdfe log_quantity log_price lakes seas1-seas12, absorb(constant) vce(r)
outreg2 using "${workdir}demand_logs.tex", replace tex

ivreghdfe log_quantity (log_price = collusion) lakes seas1-seas12, absorb(constant) r
outreg2 using "${workdir}demand_logs.tex", tex

ivreghdfe log_quantity (log_price = dm2) lakes seas1-seas12, absorb(constant) r
outreg2 using "${workdir}demand_logs.tex", tex

* report first stages
reghdfe log_price collusion seas1-seas12, absorb(constant) vce(r)
outreg2 using "${workdir}demand_logs.tex", tex

reghdfe log_price dm2 seas1-seas12, absorb(constant) vce(r)
outreg2 using "${workdir}demand_logs.tex", tex

* part d
ivreghdfe log_price (log_quantity = lakes) collusion dm1-dm4, absorb(constant) r
outreg2 using "${workdir}supply_logs.tex", replace tex

* demand and supply linear estimates
* generate interactions
gen lakes_price = lakes*price
gen lakes_collusion = lakes*collusion
gen q_collusion = quantity*collusion
gen q_lakes = quantity*lakes
gen q_lakes_collusion = q_lakes*collusion
forvalues i=1/13 {
        gen seas`i'_collusion = seas`i'*collusion
}

* run regs
ivreg2 quantity (price lakes_price = collusion lakes_collusion) lakes seas1-seas12, r
outreg2 using "${workdir}demand_supply_lin.tex", replace tex
ivreg2 price (quantity q_collusion q_lakes q_lakes_collusion = lakes lakes_collusion seas1-
    seas12 ///
```

```
                                         seas1_collusion-seas13_collusion), r
outreg2 using "${workdir}demand_supply_lin.tex", tex
test quantity q_lakes

sum price if lakes == 0
sum quantity if lakes == 0
sum price if lakes == 1
sum quantity if lakes == 1

* probit: start of price war
gen price_war_start_tomorrow = 0
replace price_war_start_tomorrow = 1 if collusion == 1 & collusion[_n+1] == 0
* drop observations where price war is already happening, so it cannot start
drop if collusion == 0
probit price_war_start_tomorrow quantity lakes dm1-dm4
outreg2 using "${workdir}probit_price_war.tex", tex

sum price_war_start if dm1 == 0 & dm2 == 0 & dm3 == 1 & dm4 == 0
tab price_war_start if dm1 == 0 & dm2 == 0 & dm3 == 1 & dm4 == 0
tab price_war_start if dm1 == 0 & dm2 == 0 & dm3 == 0 & dm4 == 1
tab price_war_start if dm1 == 0 & dm2 == 0 & dm3 == 0 & dm4 == 0
```