14.271 Glenn Ellison
Fall 2022 Tobias Salz

Problem Set #6

November 16, 2022

1. Consider a model in which a mass m of consumers are uniformly distributed around a circle of radius 1. Suppose that there are I potential entrants. Firms that enter will locate around the circle and produce a differentiated good at a constant marginal cost of c. As in the Hotelling model, each consumer receives utility $v - td - p$ if he purchases one unit of the good at a price of p from a firm located at a distance of d from his location, and has zero utility if he does not purchase the good.

(a) Let N^* be the number of firms that enter in the pure strategy Nash equilibrium I described in class. In addition to the pure strategy Nash equilibrium, the entry game also has a symmetric mixed strategy Nash equilibrium in which the firms randomize and enter with probability p^*. Show that the expected number of entrants in this equilibrium satifies $E(Ip^*) > N^* - 1$. Give intuition for why you'd guess that the expected number of entrants would be larger in the mixed strategy equilibrium and also discuss why this need not be completely correct.

(b) Work out an example to show that if firms choose whether or not to enter sequentially and if firms also commit to a location around the circle when they choose to enter then there may be pure strategy equilibria where fewer than N^* firms enter.

2. Consider a version of the standard competition-on-a-line model. A unit mass of consumers have types θ uniformly distributed on $[0, 1]$. Assume that a consumer of type θ gets utility $v_1 - t\theta - p_1$ if she purchases from firm 1, utlity $v_2 - t(1 - \theta) - p_2$ if she purchases from firm 2, and zero utility if she does not purchase. Assume first that the firms have no fixed or variable costs of production.

(a) Suppose that firm 1's product is slightly superior, $v_1 \in (v_2, v_2 + 3t)$, and that v_2 is large enough so that all consumers purchase from one of the two firms in equilibrium. What are the equilibrium prices in a game in which the firms simultaneously choose prices p_1 and p_2?

(b) Suppose that firm 2 needed to pay a fixed cost of E to compete in this market. For what values of E would firm 2 choose to pay this cost and compete with firm 1?

(c) For concreteness suppose that $v_1 = 100$, $v_2 = 94$, and $t = 12$. How does the set of values of E for which entry occurs compare with the set of values of E for which firm 2's entry increases social welfare?

(d) In the model of part (c), when firm 2 enters, firm 1 will in equilibrium serve all consumers with $\theta \in [0, \hat{\theta}^*)$ and firm 2 will serve all consumers with $\theta \in (\hat{\theta}^*, 1]$ where $\hat{\theta}^* = D_1(p_1^*, p_2^*)$. Suppose that firm 1 had the opportunity to modify its product so that it provides utility $v_1 - t\hat{\theta}^* - p_1$ to all consumers rather than having a utility that is θ-dependent. Suppose that whether firm 1 modifies its product is observed by firm 2 before firm 2 makes its entry decision. For what values of E for which firm 1 would want to make this modification?

14.271 Industrial Organization I

Fall 2022