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QUESTION 1

(a) We have a standard Cournot problem. Denote $m \equiv MC(q_1^1)$. Then, the second period profits are solutions to

$$\pi_1^* = \max_{q_1} q_1(4 - q_1 - q_2 - m) \text{ and } \pi_2^* = \max_{q_2}(2 - q_1 - q_2) - E.$$

We have the FOCs $(4-m) - 2q_1 - q_2 = 0$ and $2 - q_1 - 2q_2 = 0$, meaning $q_1^* = 2(3-m)/3$ and $q_2^* = m/3$. This gives $\pi_1^* = \frac{4}{9}(3-m)^2$ and $\pi_2^* = (m/3)^2 - E$.

If firm 2 does not enter, then firm 1 solves in the second period

$$\max_{q_1}(4 - q_1 - m)q_1$$

with the solution

$$q_1^{**} = 2 - \frac{m}{2}$$

$$\pi_1^{**} = \left(2 - \frac{m}{2}\right)^2$$

(b) If firm 1 wants to deter entry, firm 2 should earn a profit of 0 or less upon entry:

$$(m/3)^2 \leq \frac{16}{81}$$

$$m \leq \frac{4}{3}$$

This means firm 1 should produce in period 1

$$q_1^1 \geq \frac{7}{3}$$

Profits of firm 1 in both periods then are

$$(2 - q_1^1)q_1^1 + \left(2 - \frac{m(q_1^1)}{2}\right)^2$$

Note that it is not optimal to produce more than 3, because it does not affect cost in the second period, and it further lowers the profit in the first period.

So, the optimal production will be $q_1^1 \in [7/3; 3]$. Plugging in marginal cost, we have

$$\max_{q_1^1} (2 - q_1^1)q_1^1 + \left(\frac{3}{4} + \frac{q_1^1}{4}\right)^2$$

which gives solution

$$q_1^1 = \frac{19}{15}$$

[1]based on solutions by Adam Harris, Vivek Bhattacharya, and Anton Popov

Since the internal solution is to the left of the interval [7/3; 3], and the objective function is quadratic, we conclude that the solution to the restricted problem must be

$$q_1^1 = \frac{7}{3}$$

in which case firm 1 earns a total profit of 1. Note, however, that producing at the monopoly quantity 1 in the first period, firm 1 can already earn 1, and then something on top of that, if firm 2 enters, and they play a duopoly equilibrium. This proves that firm 1 will not find it optimal to deter entry.

(c) If firm 1 accommodates entry, its maximization problem is

$$\max_{q_1^1} \, (2 - q_1^1)q_1^1 + \frac{4}{9}(3 - m(q_1^1))^2$$

Again, since the marginal cost in the second period does not change if we produce q_1^1 below 1 or above 3, and moving into those regions also decreases profits in period 1, we will have optimal $q_1^1 \in [1, 3]$. Plugging in marginal cost, we get

$$\max_{q_1^1} \, (2 - q_1^1)q_1^1 + \frac{4}{9}\left(\frac{1}{2} + \frac{q_1^1}{2}\right)^2$$

which gives a solution

$$q_1^1 = \frac{5}{4}$$

(d) This is a game of strategic substitutes, and investment makes Firm 1 tough. Thus, there is overinvestment in the accommodation equilibrium relative to the open loop equilibrium, so we would expect q_1^1 to be lower than 5/4 if it could not be observed by Firm 2. We would still expect Firm 1 to produce more than 1 since lowering marginal cost is helpful in the second period to its own profits.

If firms compete on price, then we have a game of strategic complements, and investment once again makes Firm 1 tough. Then, we would have underinvestment in accommodation relative to the open loop equilibrium (i.e., when q_1^1 is unobserved).

QUESTION 2

(a) Suppose there was such an equilibrium. Then on-path firm two will always enter, as their expected profit will be $q\pi_2^D(\bar{\theta}) + (1-q)\pi_2^D(\underline{\theta})$. Given the condition on E this implies the firm will always enter. Now consider the firm in the high demand state, following not burning money, the second firm either enters or does not. The worse of the two is firm two does enter. However that would happen even if the firm burned money. Hence, the firm would prefer not to burn money.

(b) Consider the following equilibria. Firm 2 believes that so long as the stage 1 profits are above $\frac{\pi_1^m((\underline{\theta}))}{2}$ demand is high with probability 1. Further if profits are at or below $\frac{\pi_1^m((\underline{\theta}))}{2}$, then demand is low. Further, we want entry costs to be such that firm 2 would enter if they are sure it is a high demand state and not enter if they are sure it is a low demand state. For this to be an equilibrium, we need firm 1 to burn no money in the high demand state and firm 1 to burn half their money in the low demand state. This corresponds to the following expressions.

$$\frac{1}{2}\pi_1^m((\underline{\theta})) + \pi_1^m((\underline{\theta})) \geq \pi_1^m((\underline{\theta})) + \pi_1^d((\underline{\theta}))$$
$$\pi_1^m(\bar{\theta}) + \pi_1^d(\bar{\theta}) \geq \frac{1}{2}\pi_1^m((\underline{\theta})) + \pi_1^m((\bar{\theta}))$$

Finally, we need an intermediate entry cost E such that firm 2 only enters when they believe demand is high but not when they believe demand is low. This corresponds to

$$\pi_2^D(\bar{\theta}) \geq E \geq \pi_2^D(\underline{\theta})$$

In the Fudenberg-Tirole model, the signal jamming leads to an increase in production and thus an increase in social welfare. In contrast, in this model, the signal- jamming leads to welfare losses.

QUESTION 3

(a) Controlling for firm-fixed effects and average stack market trends, firms in competition and not in competition with the leveraged firm would have had parallel trends in their stock market returns.

(b) Chevalier assumes that the firms in competition with the leveraged firm all have the same response, and all firms also not in competition also have the same response. She interacts x_{ij} and $1 - x_{ij}$ with D_{jt} to separately estimate the impact on competing and non-competing firms. Alternatively, she could have used the market share of competing firms, rather than a binary 0/1.

(c) Doing such a comparison would test whether competing and non-competing firms have differing responses to an LBO; Chevalier only tests whether competing firms have a non-zero response. If she were to perform a significance test on the differences, she would have all null results.

QUESTION 4

(a) Given the functional form, the utility for an individual of type $\theta \in \left[0, \frac{1}{2}\right]$ is

$$u_\theta = \begin{cases} -p_1 & \text{buy from 1} \\ -2t(1-2\theta) & \text{buy from 2} \\ -2(v - t\theta) & \text{do not buy} \end{cases}$$

If we assume that v is high enough such that all consumers buy from a firm (sufficient to assume $v > \frac{t}{2}$). If $p_1 > p_2$, the consumer who is indifferent between buying from 1 and buying from 2 is located at

$$-p_1 = -2t(1 - 2\widehat{\theta}) \Longrightarrow \widehat{\theta} = \frac{1}{2} - \frac{p_1 - p_2}{4t}$$

All consumers with type $\theta \in [0, \widehat{\theta}]$ will purchase from firm 1.

(b) Solution: Given firm 2's price, firm 1 maximizes

$$(p_1 - c)\left(\frac{1}{2} - \frac{p_1 - p_2}{4t}\right)$$

which occurs at

$$p_1^* = \frac{1}{2}(c + 2t + p_2^*)$$

Setting $p_1^* = p_2^*$ yields a symmetric equilibrium with $p^* = c + 2t$, implying that firms are able to incur more rents than in the Hotelling model(intuitively– the consumers are "more captive").

QUESTION 5 BONUS-NOT NEEDED

(a) Let x_1 be the distance between the indifferent type and firm 1. Then their distance from Firm 2 is $1/4 - x_1$. The indifferent consumer has:

$$v - tx_1^2 - p_1 = v - t\left(1/4 - x_1\right)^2 - p_2 \Longrightarrow x_1 = \frac{1}{8} + \frac{2\left(p_2 - p_1\right)}{t}$$

The demand facing Firm 2 is then $2\left(1/4 - x_1\right)$, since Firm 2 has consumers on each side of them. The demand facing Firm 1 is $2x_1 + \frac{1}{2}$, since Firm 1 also receives all captive consumers on the other side of the circle (assuming v is large enough as we usually do). Thus, profit functions (ignoring the entry costs) are given by:

$$\pi_1 = (p_1 - c)\left(3/4 + \frac{4\left(p_2 - p_1\right)}{t}\right)$$
$$\pi_2 = (p_2 - c)\left(1/4 + \frac{4\left(p_1 - p_2\right)}{t}\right)$$

The first order conditions are then as follows (noting that with linear demand curves the second order condition is always satisfied):

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Longrightarrow 3/4 + \frac{4p_2}{t} - \frac{8p_1}{t} + \frac{4c}{t} = 0$$
$$\frac{\partial \pi_2}{\partial p_2} = 0 \Longrightarrow 1/4 + \frac{4p_1}{t} - \frac{8p_2}{t} + \frac{4c}{t} = 0$$

Solving these yields equilibrium prices $p_1^* = \frac{7t}{48}$ and $p_2^* = \frac{5t}{48}$

Plugging this back into the profit equations and subtracting out the variable costs yields profits $\pi_1 = \frac{49t}{576} - 2E_1$ and $\pi_2 = \frac{25t}{576} - E_2$.

For the intuition, label the 2 locations of firm 1 as 12,6 and firm 1 at location 3(as on a clock). When firm 2 prices they have a competitor firm 3 units away clockwise and 3 units away counterclockwise. Whereas when firm 1 prices for the 12 location they have a competitor 3 units away clockwise and the only other location is one of their own shops 6 units away. Not only is this own shop further away and thus less likely to steal their consumers, but this is one of their own units and thus has less incentive to do so. This causes firm 1 to set higher prices.

(b) When $N = 1$ the indifferent type is debating between the outside good and buying the good. Call this type x^* and we have

$$x^* = \sqrt{\frac{v - p_1}{t}}$$

The firm will sell to $2x^*$ consumers since they sell to consumers on both types of the circle. Profits are thus

$$2p_1\sqrt{\frac{v - p_1}{t}}$$

which can be solved using a computational solver to give an optimal $p_1 = \frac{2v}{3}$. If this still gives positive utility to the type that is exactly $\frac{1}{2}$ away the FOC is not sufficient since can increase prices to the point where this type doesn't buy. Since this is simpler, we will solve for this case. One can check this is equivalent to $v \geq \frac{3t}{4}$. When this is the case they charge a price of $v - \frac{t}{4}$ and get a profit of $v - \frac{t}{4} - E_1$.

If instead $N = 2$, we want them to sell to all consumers and charge them the exact indifferent price of $v - \frac{t}{16}$. If it was optimal to bind the participation constrains of all agents when $v \geq \frac{3t}{4}$ then it will be optimal to do so as well when there are 2 firms as well since we will have the same FOC, but now the indifferent type is closer to the firm and thus we are even more likely that the price given by the FOC is too low given that this consumer is closer. Solving a similar condition to the above yields a condition that $v \geq \frac{3t}{16}$. The problem said we want the firm to prefer having only 1 station as opposed to 2 absent entry deterrence motives so we need:

$$v - \frac{t}{4} - E_1 \geq v - \frac{t}{16} - 2E_1 \iff E_1 > \frac{3}{16}$$

.

We also need that firm 1 prefers having two units to shutting down and not competing at all. This is equivalent to $2E - 1 \leq v - \frac{t}{16}$.

We also need that firm 1 prefers being with 2 units than being with 1 and competing with firm 2. If there is entry, we have that Firm 2 will simply locate across from Firm 1 . This will yield an indifferent consumer that is a distance $x = \frac{1}{4} + \frac{p_2 - p_1}{t}$ from Firm 1 . Using symmetry, we can calculate $p_1 = p_2 = \frac{t}{4}$. This yields profits for both firms of $\frac{t}{8} - E_1$ This implies that $v - \frac{t}{16} - 2E_1 > \frac{t}{8} - E_1 \implies v > \frac{3t}{16} + E_1$.

The final thing we need is that firm 2 will enter if firm 1 only has 1 location, but won't enter if they have both. We know from (a) what the profit is if they are competing against 2 locations and we solved above what the profit will be if they are competing against one location. Comparing these two necessitates that $t < \frac{576}{25}E_2$.

Accumulating all our constraints has

$$\begin{aligned} v &\geq \max\{\frac{3t}{4}, 2E_1 + \frac{t}{16}, \frac{3t}{16} + E_1\} \\ t &\in (8E_2, \frac{576}{25}E_2) \\ E_1 &\geq \frac{3t}{16} \end{aligned}$$

One can check that if $t = v = 1$ the first constraint simply says E_1 cannot be too large while the third says it cannot be too small. A value slightly above $\frac{3}{16}$, such as $\frac{1}{4}$, works for E_1. Then since $\frac{576}{25} > 8$ we can find a value of E_2 that works. One value is $\frac{1}{16}$.

(c) If firm 2 enters there this will cause bertrand competition down to a price of 0 at this point. Now when firm is pricing at the 12:00 point there indifference consumer is such that $x^* = \frac{1}{4} - \frac{p}{t}$ which gives them a profit of $2px^*$. Solving this yields $p^* - \frac{t}{8}$ and a profit of $\frac{t}{32}$.

If they do withdraw their point then we calculated the profit in part (b) which yields a profit of $\frac{t}{8}$, so firm 1 would prefer to withdraw.

This implies entry deterrence through brand proliferation is possible only if they can feasibly commit to keeping their locations.

14.271 Industrial Organization I
Fall 2022