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Fall 2022 | Tobias Salz

Problem Set #8

Due December 7, 2022

1. Consider a model of duopoly competition between firms selling undifferentiated products to boundedly rational consumers. There are a continuum of consumers of unit mass. Each consumer will get utility $v - p$ if he buys one unit of the good from either firm at price p and utility 0 if he does not purchase the good. (Assume consumers never buy more than one unit and never buy from both firms.)

Assume that firms have a constant marginal cost of $c < v$ and must choose prices in the interval $[c, v]$. (This would happen in equilibrium if boundedly rational consumers can't be made to pay more than v, but it's simpler to assume it directly than to complicate the description of the boundedly rational consumers.)

Assume that the firms simultaneously choose their prices p_1 and p_2 as in the Bertrand model, but augment the model by assuming that prices can be stated in two different ways: A and B. (This could be quoting an all-inclusive price vs. a price that includes many hidden fees; or quoting the price in dollars vs. Euros; etc.)

Assume that a fraction α_1 of consumers are extremely naive. They are able to identify and buy from the lowest-price firm if both firms use method A for stating prices. But otherwise than cannot tell which price is lower and are equally likely to buy from both firms.

Another fraction α_2 of consumers are somewhat naive. They buy from the lowest price firm if both firms use method A or if both firms use method B. But if the firms use different methods for describing their prices, then they also purchase one product at random.

Finally the remaining $1 - (\alpha_1 + \alpha_2)$ consumers are rational and buy from the lowest-priced firm.

(a) Suppose that both firms use method B to describe their prices. Derive a symmetric mixed strategy Nash equilibrium that would describe how the firms would choose prices.

(b) Consider an augmented game in which the firms simultaneously choose both their prices and the method in which they present their prices. Show that this model cannot have an equilibrium in which the firms present their prices in the same way.

2. Consider a Butters-style model of advertising by a monopolist. A continuum of consumers of unit mass have values $v_1 \sim U[0, 1]$ for firm 1's product. Firm 1 has no production costs. It does, however, have advertising costs. An advertising campaign that will inform a fraction $x \in [0, 1]$ of consumers about the existence of firm 1's product costs cx^2. Assume that uninformed consumers never buy from firm 1 and informed consumers buy if and only if $p_1 \leq v$.

(a) Holding x fixed what price p^m will the monopolist choose and what will its profits be? What advertising level x^m does the monopolist choose? Does Butters' result about the advertising level being socially optimal carry over to this model? Can you provide some intuition for why it does or does not carry over?

(b) Consider now an asymmetric duopoly model. Assume that firm 1's advertising technology is as above. Assume that the consumers' utilities for purchasing from firm 1 are also as above *if* they don't buy from firm 2. If a consumer buys from firm 2, however, then he or she has no need for firm 1's product.

Assume that firm 2 has a superior product and a superior advertising technology. All consumers get utility $v_2 = 1$ from firm 2's product and all consumers are aware of firm 2's product.

Consider a two-stage game in which firm 1 first chooses x (which is observed by both firms) and firms 1 and 2 then compete by simultaneously choosing prices p_1 and p_2. What values of p_1 and p_2 must be chosen in the second stage after firm 1 chooses the equilibrium level of x?

(c) What are the firms' profits in this equilibrium (as a function of x)? What advertising level would firm 1 choose?

3. An environmental group which wants to build a wildlife preserve is bidding against a logging company in a government auction for a tract of land. Because the two parties will use the land for different purposes, assume that the auction can be modeled as one with independent private values. Assume that it is common knowledge that the environmental group's valuation for the land, v_e, is drawn from a uniform distribution on [0, 2], and that the logging company's valuation, v_ℓ is drawn from a uniform distribution on [1, 2].

(a) Find the equilibrium strategies and the probability with which each party wins the tract in a second price sealed bid auction. What is the government's expected revenue? Would you expect a first price sealed bid auction to raise more, less, or the same amount of revenue (in expectation)?

(b) In what ways is the asymmetric auction model described above very different from that used by Hendricks and Porter to model the asymmetries between firms in bidding for offshore oil drainage tracts?

(c) Suppose that the seller also has a second unit to sell. It is common knowledge that player 2 would get no incremental benefit from winning a second unit. Player 1 would get the same benefit from the second unit that he gets from the first (what this is remains private information and uniformly distributed on $[0, 2]$.)

Consider a multiunit ascending auction in which the two bidders are asked to raise two hands if they want 2 units at p and one hand if they want one unit at p, with bidding ending as soon as only two hands are raised.

Find a perfect Bayesian equilibrium of this game. What is the seller's expected revenue? What is the probability that the goods are allocated efficiently?

4. Consider an $N + 1$ bidder asymmetric IPV auction model. Suppose that N bidders have values uniformly distributed on $[0, 2]$ and one "high-value" bidder has a value drawn from a uniform distribution on $[1, 2]$.

(a) What are the equilibrium bidding strategies in a second-price sealed bid auction assuming bidders do not use weakly dominated strategies? What is the probability that the high-value bidder wins the auction?

(b) What is the seller's expected revenue in the auction?

EXTRA PROBLEM ON LATER MATERIAL – NOT DUE

5. Consider a variant on the Edelman-Ostrovsky-Schwarz model of Google's advertising auction. Suppose that there are three firms with per-click values v_1, v_2, and v_3, drawn indepedently from some distribution F. Suppose there are two slots for ads. Assume that the number of clicks that firm j will receive while in position i is $\alpha_i w_j$, where $\alpha_1 > \alpha_2$ and w_1, w_2, and w_3 are potentially different.

(a) What is the socially efficient allocation of advertisers to slots?

(b) Suppose Google wanted to allocate slots using a VCG mechanism. What would this mechanism look like in this environment, e.g. what would firms announce, and what payments would firms make?

(c) Suppose that w_j is known only to firm j and that Google allocates slots via a simple unweighted GSP auction. Give an example to show that the equilibrium outcome can be different from the social optimum.

(d) Suppose that Google uses a weighted GSP auction using the w_j as weights, i.e. the firm in position i makes a per-click payment of $b_{i+i} w_{i+1}/w_i$. Derive the formula for the optimal dropout strategy of firms in an ascending bid auction. Do these strategies imply that firms will drop out in the efficient order?