

15.060 Data, Models, and Decisions
Homework 2
Due: MONDAY, September 22, 2014

1. The amount of yogurt sold by Clover each day (for their lovely yogurt parfaits) obeys a continuous uniform distribution between 400 ounces and 600 ounces.
 - (a) What is the probability that tomorrow Clover will sell between 450 and 500 ounces of yogurt?
 - (b) What is the probability that tomorrow Clover will sell between 300 and 350 ounces of yogurt?
 2. Assume that the net revenue per day for a certain e-store is approximately obeys a Normal distribution with mean $\mu = \$10,000$ and standard deviation $\sigma = \$6,500$.
 - (a) What is the probability that tomorrow's net revenue will exceed \$9,500?
 - (b) Suppose that the probability that today's net revenue exceeds the value b is exactly 0.60. What is the value of b ?
 - (c) What proportion of the days does the store have negative net revenue?
 3. Your company melts bars of gold to produce liquid gold. The daily production of melted gold approximately obeys a Normal distribution with mean $\mu = 50$ tons and standard deviation $\sigma = 10$ tons. Compute the quantiles of the daily production of melted gold. More specifically:
 - (a) If the probability that daily production is less than a tons is 0.25, then what is a ?
 - (b) If the probability that daily production is at least b tons is 0.25, then what is b ?
 - (c) If the probability that daily production is at least c tons is 0.50, then what is c ?
- And here is a slightly more challenging question:
- (d) What is the probability that daily production will be either more than 70 tons or less than 30 tons? (In other words, what is the probability that our Normally distributed random variable will be least 2 standard deviations away from its mean?)

4. JoAnne has gathered data on stock prices and rates of return for companies A and B, and has converted this data into the joint probability distribution of rates of return for the two companies shown in Table 1.

Probability (p_i)	Annual Rate of Return of Asset A (%) (x_i)	Annual Rate of Return of Asset B (%) (y_i)
0.04	5.1	6.9
0.09	5.3	6.1
0.16	7.8	8.3
0.19	8.2	9.2
0.17	7.7	7.9
0.15	9.0	7.1
0.10	10.7	9.3
0.10	11.1	11.4

Table 1: Joint probability distribution of the rates of return of companies A and B.

- (a) Compute the mean, variance, and the standard deviation of the rate of return of company A.
- (b) Compute the mean, variance, and the standard deviation of the rate of return of company B.
- (c) Compute the covariance and correlation of rates of the return of companies A and B.
5. The website <http://HelpingHomeowners.com> offers a variety of information to assist homeowners in repair and maintenance. The site features banner display ads of local service companies and local home product companies, for which HelpingHomewners receives ad revenues based on click-thrus. Let X denote the daily number of click-thrus of display ads of homeowner service companies on the site, and let Y denote the daily number of click-thrus of display ads of home product companies on the site. Suppose that $\mu_X = 254.1$ and $\sigma_X = 88.7$, and $\mu_Y = 177.5$ and $\sigma_Y = 85.2$. Suppose further that $\text{CORR}(X, Y) = 0.63$.
- (a) Suppose that revenue to HelpingHomewners from click-thrus is \$0.04/click-thru for service ads and also \$0.04/click-thru for product ads. Compute the mean and

standard deviation of total daily revenues. Compare the relative risk associated with revenues from service companies, revenues from home product companies, and total revenues. What do you observe?

- (b) Suppose that revenue to HelpingHomewners from click-thrus is \$0.03/click-thru for service ads and is \$0.07/click-thru for product ads. Compute the mean and standard deviation of total daily revenues. Compare the relative risk associated with revenues from service companies, revenues from home product companies, and total revenues. What do you observe?

6. You are tasked with constructing a portfolio out of three assets: A, B and C. The annual returns, risk, and correlations between the returns on these assets are given in Table 2. The requirements of the portfolio are: (i) it should be composed of exactly two assets, (ii) the funds should be invested 50% in each asset, and (iii) the expected annual return of the portfolio should be at least 10%.

Asset	Annual Expected Return(%)	Risk (%)	Correlation		
			A	B	C
A	13	13	–	-0.20	-0.15
B	7	12	-0.20	–	0.30
C	9	15	-0.15	0.30	–

Table 2: Annual returns, risk, and correlation between the returns of assets A, B, and C.

- (a) Find the portfolio that satisfies all of requirements and has the minimum risk.
- (b) [Challenge] Can you find a portfolio with less risk if you can invest any percentage in each of the two assets (while still satisfying the minimum 10% expected return requirement)?

MIT OpenCourseWare
<http://ocw.mit.edu>

' DND 0 RGHV DQG' HFLMRQV
Fall 2014

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.