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PROFESSOR: So let's get started. Today we're going to be talking about preflop, which is the last thing I definitely want to cover before the [? Sakuna ?] tournament. As you know, this is going to be one of my last lectures. I'm teaching one or to over the next two weeks, but we're primarily going to have guest speakers talking about more the macro poker environment.

So why are we doing preflop? So in tournaments, most of your value is going to come from what you do preflop-- playing it close to optimally. And the reason is because a lot of people don't do this good at all. Like they really, really play preflop badly because it's very counter intuitive. Especially live. Like people have a much tougher time doing this because either they're afraid of getting knocked down on a bad hand, or their afraid of showing down a bad hand, or live players are just worse in general.

So for whatever reason, people screw this up online. A little bit on live a lot. In addition, one of the reasons that we're spending an entire day on it is it's relatively easier to solve from a mathematical standpoint. Like we get close to a Nash equilibrium because there aren't that many variables. Whereas postflop, there are a million variables.

It's more related to kind of putting things into patterns that you might be close to. So that's why I'm doing this. And then let's start with a scenario that we're going to be analyzing for the rest of the class. There we go. So here's our scenario. So I'm-- how it works heads up is the dealer button is a small blind. That's a slight break in the rules because you assume I'm button, he's small blind, I'm big blind, but they change around a little bit. This way, the button isn't the last one to act every round.

So here, I'm first one to act preflop and then last to act every round thereafter. If I were big blind, I'd be less every single round. So it's a minor variation that you guys should just know.

So situation-- I'm small blind for 125, he's big blind for 150, and there's a 25 ante, which is why there's 50 in the pot. And the question is what do we do here? OK, so we have 9-6 offsuit with 2 1/2 m. We're in the small blind, and we're trying to figure out what do we do.

So the answer here might not be that intuitive, and I don't think I'd give it to you right away. But how we figure it out is just with a normal semi-bluffing equation. We take a look at-- OK, so everything preflop is a semi-bluff because we have some chance of winning, and we're virtually never less than 30%. So we just use this semi-bluffing formula that we've seen before where our EV is just going to be the pot times the chance he folds, plus the chance he doesn't fold times our EV when he calls.

We need to figure out some sort of calling range. I just made up one here that seemed like about at the pro level, and this is pretty wide. Like him calling with like Ace-2 or Jack-10 or 2-2. This is a wideish range. I think a lot of players for the tournament life might not call this wide, but it ends up being like 27.6% of hands he's calling. So we're going to use it as a baseline, and then later we'll show why that doesn't matter.

So our question is, what's our equity versus range here? And what we can do there is just plug into PokerTracker. So the idea is we don't know what hand he has and we don't care. It's unrealistic to think that we can get down to any sort of hand he'll be calling with, but we can get down to a range.

We can say he's equally likely to have any assortment of the hands in that range because if he calls, that means that he has one of those hands. So we can compare our equity versus any one of those hands to get our equity versus range, and we can do that in PokerTracker.

So what you're going to do is you're going to open up the equity calculator, you're

going to put in 9-6 off, and then you're going to put in this. I actually just drew it in there, but that's the same thing I showed before, which is pocket 2s are better. Ace-2, King-2 , Queen-- sorry. King-10, Queen-10, Jack-10, and then the same for offsuit, which is the representation of this entire thing. I just need to separate offsuit or suited, and then I assumed by jack-10, it meant that entire corner, although PokerTracker likes specifying.

Anyway, so this is our equity versus. This is saying that if we go in with 9-6 and he calls with anything in that range, we are 34% to win this hand. So I don't know if that's about what you'd expect. It seems higher than I would have thought initially, but that's our equity verses range. If we go all in and he calls, we can assume we're a 35-65 underdog without even knowing what he has. And then once we find out what he has, we will find out whether we're slightly better or worse, but that doesn't really matter.

So we can calculate our EV in this hand in the same way that we're used to calculating semi-bluffing. The EV of the push is going to be this. The chance of him folding times 425. If he calls 27% of the time, that means he folds whatever this is like 73% of the time. So that's our value from him folding-- the fold equity. And then 20% of the time we're in a showdown situation where of that [? push ?] time that he calls, we're 35 to win 1250 and we're 65 to lose 950, resulting in a total equity of 253.

So that's a lot of chips for what might seem like a very marginal move. By going all in here, you're actually making that many chips. And in fact, a more provocative way to describe it is if you don't do this, you are losing 250 chips. You only need 1,000 to win this whole tournament, and by folding what seems like a very weak hand in this position, you are just giving him 250 chips of value.

So that's a lot, and that shows how counterintuitive this is. That a situation where you have really bad cards, you don't realize that these are actually really, really good cards in the situation. In fact, no matter what, your cards are basically good in this situation. Because this situation makes any two cards good enough. So let's talk

about-- so we made one assumption, which is what his call range is.

Did you have a question? Oh, OK. So we made some assumption about what his call range is, and I said it didn't matter. Why? Because what we can do here is make the call range a variable. So this fold percent is a variable, and then our win percent is related to the percent that he folds. Because if he calls a small range of hands when we get called, he's going to be crushing us. Whereas if he calls like 90% of the time, we're probably ahead of his range because he calls a lot of worse hands.

Anyway, so you can take a graph and just look, OK, he calls somewhere between 0% of the time in 100% of the time, and then we can do this EV equation for all those hands by calculating OK, so what's our equity versus calling range? This doesn't change. We still win or lose the same amount every time. And say so if we were always pushing here, if he calls with whatever range, this is our EV.

And then one thing that's interesting about this graph is what? It's always positive, which means that no matter what his calling range is, this push is good. It might be more intuitive this way. Like if he folds 100% of the time-- if he only calls 0% of the time, we trend up to a number around here. And what number's around here?

That's just this. It's the pot before we do anything. So if he folds 100% of the time, we just win the pot. Like that's our EV every hand, and it just trends based on this. Where the more he calls, typically the better the worse our EVs going to be. Yes? [? Yes. ?] We'll get to that. And that's a good question, although it's going to take us half the class to answer that.

Anyway, so that's a cool thing here. So it's always positive, which means I'm saying-- no matter what his-- the villain does, always, always push all in with 9-6 offsuit in this position.

So let's break it down into components. So to explain intuitively what drives this graph is your equity there is split into two parts because it's a semi-bluff. It's your fold equity and your showdown equity. So let's turn this into lines.

So you see your fold equity just decreases the more he calls in a linear fashion. So

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that should be fairly intuitive. The proportion he calls reduces your fold equity by that amount, and then your showdown equity has a little curve here where you get more value from the showdown on average the more he calls.

The only reason there's a little bit of curvature is because it's multiplied by the chances of him calling in the first plAce. So it's tilting in some direction. But anyway, these are curved. Makes this a very interesting optimization puzzle. And the reason we're positive at the end is for this reason.

We're basically-- so this line, the total equity, is just the sum of these two. Where our showdown value is going to be negative over basically this entire range. Even when our fold equity reaches zero, that's when our showdown equity creeps just above zero.

Regardless of what his calling range is, your average is like 175 chips of EV. Like the 20% call-- or whatever we said. The 27% calling range isn't necessarily optimal. He knows you have these cards. But on average, you're getting-- whatever, like 100 and something chips. And just to show how bad is folding, if you just open fold this for some reason, that's as bad as purposely calling it all in with 3-4 versus Ace-King. Like that's an equivalent amount of EV loss as folding this hand.

So now let's talk about how hard this is going to be. So push/fold decisions are really hard to kind of intuitively figure out just because of the curvature and the steepness of those graphs are moving in weird directions and they're both moving at the same time with regard to all the other variables. So it makes it so that it's very hard to come up with very quick rules.

Some variables are going to affect our decision, and the result is either push or fold. Our result isn't even as complicated as betting a certain amount, but the variables you have to consider are our cards, our position, our stack, and the villain's call range, which results in this five-dimensional ray that we have to slice and dice so that we have our push/fold decision range.

So what we want to do is isolate for specific things. We want to say OK, let's take--

let's make this and this static so that we can solve for these two, or let's make these two static so we can solve for these two. So we can end up with a chart that looks something like this. And this is a little bit more manageable. We can just say this green area is when you should push and this yellow area is when you should fold.

That's our goal here. To be able to develop that sort of chart. And then figure out what's the best thing to isolate. So let's get to it.

So range is the postset of hands. This is how you write it. This is the assumption that we're making. So we're doing it for two reasons. Analyzing our opponent, which we're not going to be doing. We're going to assume we have no information about him because it's preflop, and a lot of it-- one of our assumptions is that he hasn't even acted yet, so we have no information at all.

But we're going to be using it to determine our plays. Basically we're saying we have some sort of decision [INAUDIBLE] for our range. Like we certainly can't solve this for every single hand or at least we can't remember it, so what we're going to do is come up with a line where we say every hand above that certain line, i.e. That range above that line, is going to be good for our specific decision.

And we're bringing these into percentiles, and that's how we're going to describe them. And we're using Sklansky-Carlson. His rankings for our breakdown of percentiles. But here's his original question, and this is just to explain where these come from and why this is particularly relevant to what we're doing.

So his question is you're the small blind and you have a choice between either folding or open pushing a hand, or open pushing here means you push and you show your hand before he makes a call. And the question is how many chips is this good for? If you have one chip, or I guess we have two chips, you have since they're 1-2 blinds, if you have two chips and one is already in the small blind, your pot odds are 25%. So even if he knows-- he's going to call if he's anything more than a 50% favorite, so you have the odds to do that with at least a pretty bad hand and a lot of good hands.

So x here is how big can your stack be before this starts becoming unprofitable? And the answer for Ace-Ace offsuit is about 70, and to explain how we got to that, we're going to make a couple assumptions. And you can see the clear relation between this methodology and what we're doing.

So 1-2 blinds-- here a small blind with whatever. Here are bets x all in. And we're going to say big blind's going to call when EV's more than 50. Which he might do pot odds, which we're just going to ignore now for simplicity. We're just going to say big blind is calling when he's more than 50% to win. It's actually zero when he has the pot odds, which is this equation, but we're going to forget about that.

Let's just say he's going to only call with literal better hands. So here he's going to call with Ace-8 or pocket 2's because we have Ace-8. He's going to call with a pocket pair, which is always going to be better, and Ace-8, which is going to be equal or better. So that's his calling range 13% of the time.

So we win 33% for this range, which is you just plug-in to the PokerTracker and it'll give you that you're a 30% underdog, which is what I did here. That's how you do it. And we get our EV from the situation, and we have to solve for x. So we want EV to be 0 because we're finding the marginal EV, and this is what we get.

We get x, which is all the chips we had before we paid the big blind, and we lose x minus 1, which is all the chips we have after paying the small blind. So a little bit of nuance there and only changes our number by one, but that gives us our break even. And our break even here is like 62x. So the reason it's a little bit lower is it doesn't factor in the big blind doing pot odds, but that's the idea.

So what we're going to do is we're going to solve this and we're not going to do it. Someone else did it. We're going to solve this for every single hand, and this is what Sklansky did. So he figured what your number here is going to be two things. The number of hands you are slightly better than, and why is this factor important? What's he going to do it if you have a better hand than him?

He's going to fold. So this determines our fold equity, and then chance of winning

when you're behind determines our equity when he actually does call. He just has perfect information which is why it's a little bit of a change and we're able to solve it out perfectly here.

So Sklansky didn't know how to program, which is why Scott Carlson, or Victor Chubukov a couple days later-- apparently later the same day- came up with the answer. Where he said Aces have infinity value here where you can have an unlimited amount of chips and then show Aces and push them, and it's plus EV. We're not saying it's the most optimal decision, but you won't actually lose chips on average even if you have an unlimited chip stack. Whereas King, it only works for 1,290 chips.

So we can solve it for every hand to get down to, you could do this with 3-2 off with 1.8 chips and so on. So this is our primary way to rank hands, and we're assuming that at least in a heads-up situation, this is a pretty good idea of what hands will have the most value especially when it comes to going all in.

One other method is going to be equity versus three randoms, and it might be more relevant for multi-way pots. You just rank them by their expectation against three callers who haven't looked at their hand. But we're not going to do that especially because we're assuming generally we're going to get one caller, especially heads-up where you can only get one. But we're going to expand it out later.

So just an example of what these are. Top 1% is Aces, top 5% is 10s and Ace-Queens. 30% is this. And sorry to break it to you guys, but you're going to have to memorize these. So 5% you just remember their premium hands, just ten or Ace Queens, and I'm giving you little mnemonics to help you remember it. Like Ace-10 or better is 10%, and then like some sort of pocket pairs-- it's not even that big of deal if you go down to 2s here.

20% is Ace-2 or pocket 2's. That's how you're going to remember it. And then 30% is Broadway, so Broadway is any two fAce cards. And an intuitive way to think about it is based on this. Broadway is just this corner here, which is about 1/3 of the graph when you count in pocket pairs.

So that's how you just remember 30%-- it's that corner. So it's not that bad. Just remember top 20% is Ace-2 and 2s. 30% is Broadway, and then 10% is Ace-10-- not that bad. So 50% is just going to be the diagonal on the graph. It's any two cards adding up to 15.

This is what I mean by the diagonal. Like this is your 13-by-13 graph where these are offsuit and these are suited, which is why we double count them. And then this hand adds up to 15. Ace-two and so does King-3 and so does Queen-4. So this is your 50th percentile hand. So that's how you're going to remember that.

And then you don't really need to remember anything more than that. So 100% is going to be any two cards. It's going to be the entire graph here.

So we're going to be talking about-- yeah? Percentage is the percent of hands that that range represents. So like this is the top 10% of hands. So if he calls-- if we assume he's going to call 10% of the time, we are saying that he will call with these cards and these cards only. And we're going to be talking about ranges from now on because it helps us understand what cards he has, to some extent, but also lets us use it as an idea to get-- to figure out exactly how often he's going to call.

So we're just going to be talking about ranges, and this is what I mean when I say ranges. And to the extent that you're just doing this at the table, you just have to memorize these three numbers. Because this is easy, this is easy, and then just remember that tens are our 5%.

When we're talking about ranges in general, a plus EV range-- a decision that's good for a range means that on average, your decision is profitable for every hand for that range in general. But an optimal range is profitable for literally every hand. So you can do a lot of things that are profitable for any two cards, but 3-2 off might actually not be profitable.

So it's not necessarily optimal. You want to make sure that every [INAUDIBLE] you can, every single card in that range is optimal and is profitable, which gives you the optimal range. It's the most plus EV range for that type of decision.

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An example of this is-- so say you're playing against Ace-Queen and you call with this range. Pocket 5's and Ace-10 or better. So you are ahead of this [INAUDIBLE] Queen because you're 53% to win the hand with this range. However, we know it's not optimal because we're calling with two losing hands-- Ace-10 and Ace-jack. So a more realistic, more optimal range is going to be pocket 2s and Ace-King or better.

So that's going to be the difference there. So I want to make sure we're not solving for something that's a slight favorite when actually missing out on something a little bit more optimal. Here we go, so this is a better range. 5-5 or Ace-Queen, actually 2s or Ace-Queen would be a little bit better, and we win 60% of the time.

And then if a range is optimal, then we know for every hand in that range, we have a plus EV decision. And that's why this is the action we're going to use to prove that if we solve for a range, then we know that if you have any card in that range, our rule is still good for it. So that's why I'm showing you that.

Right, so that's it. So that's all we're going to do in terms of defining what ranges are and how we got those numbers. And from now, we're going to talk about making decisions based on a range.

So let's talk about preflops. Our assumption is hero has M less than 10. We're in that period of the tournament where your M is not going to be that high. The villain is calling some percentage of hands that are presumably the top whatever of hands. We're guessing he's not calling with a worse hand then-- and folding better hands. [INAUDIBLE].

He might be a little bit-- his view of what are good hands and bad hands might be a little bit off of ours, but we're just assuming everyone has the same. [? ICM ?] doesn't matter, so we don't care about payout in tournaments. We're just trying to maximize our chip-- our chip EV, and we only have two decisions-- push or call.

So the way that we're going to come up with this rule here for heads-up is-- so what we're doing is we want to figure out what our push/fold range is for heads-up in every scenario. And the way that we're going to do this is first, we need an equation

that tells us range for range equities. So everyone can figure out Aces versus whatever is like 80-20, and then 2 over cards with a pocket pair is 50-50ish.

But we want to know what's a range versus a range. Like if we push 70% of the time and he calls 30% of the time, what does that translate to in terms of our equity? And it ends up being like 60/40, but we want to get an idea of what that trend is because that's going to materially change our ability to come up with an optimal solution here.

So we want to build a table of range versus range, and then we come up with a formula that will let us put in two ranges and come up with an equity calculation for both of those ranges. Then we're going to develop an EV model for semi-bluffs, which we already did, so that should be quick. For each m, find Nash equilibrium if one exists.

So a lot of people-- if you Google "Nash equilibrium" for a preflop, you'll find something, it's wrong, and later I'll show you why. And therefore, unstable equilibriums-- it pretty much always is. Find a reasonable range and figure out what kind of assumptions we need to make to make our push/fold decisions correct.

So let's do this first thing. So if we just-- for each-- say that we use a hero's range of 50%. So in PokerTracker, we put top 50% of hands. And then for each range of hands for the villain, we calculate the equity for that. And we do that for whatever I did, like 15 different ranges.

So if we push 50% of the time and he calls 50% of the time, what's our equity? What's our chance of winning? 50, right. So we have an even equity because we are pushing and calling the same types of hands on average.

Whereas if he calls 100% of the time, our equity's actually closer to 60. We are the favorite-- we are a 60/40 favorite on that hand. Whereas if he calls with just Aces, then we are like 17% to win the hand. That's the idea.

So our goal here is to come up with some sort of equation that we can just plug-in

to-- we can do math on. Like we want to be able to do calculus on it, so we need an equation. So what type of function does this look like because we want to try to fit a curve to it.

Exactly. It seems like a logarithmic function and within r squared of 99, it says that that's the logarithmic equation for it. So that works when the range is 50%, but let's take a look at when we change the range to 30%. So what this is saying is we are pushing 30% of the time and he is calling x percent of the time. And what this line is our chance of winning when he does that, and this is also logarithmic.

So we're seeing a little bit of a pattern, and then it also has a really good r squared. And then if we push 10% of the time, it still like, OK, it's 98 and change when we look at our equities versus his calling range.

But then it starts to get bad. If he calls 5% of the time, our r squared when we fit a log normal function, or a logarithmic function, is only 95. And if he only calls 3% of the time, our r squared is 89. Like this is not really lot logarithmic when we start getting in very tight ranges. And in 1%, it sucks. It's not even close.

So what we're trying to do is develop an equation for figuring out range versus ranges, and we see that logarithmic works some of the time. So the reason it doesn't work, just to give you kind of an idea why it doesn't work, is because the top 1% of hands is three hands. It's Aces, Ace-King, and Kings. So it's materially changing based on whether he's-- if we push Ace-King, whether he calls with only Aces or Kings or also Ace-King jumps us between here and here. So we have huge gaps when it comes to very tight percentages, so that's why we break a little bit up here.

And when we turn it around, we have the same type of thing. This is looking at OK, say that we know he's going to call 50% of the time. If we push x percent of the time, what is our chance of winning? So if we 100% of the time and he calls 40% of the time, then we are 40% to win.

So that's what this chart is telling us, and you see the same type of pattern. Where

50% range versus this kind of range is good. If we push whatever and he calls 30% of the time, it's still good. But then if he calls 2% of the time, it starts becoming bad.

So what are we taking away from this? And the whole plan is we want to come up with an equation that just gives us these numbers. I only got these from pushing them into PokerTracker and were trying to fit a certain equation onto it. So our takeaways here is that like range versus range relationship is probably logarithmic. That seems to be a good estimate, but it's definitely not good in the top 5%.

So with regard to our model, we're just going to say this is probably not that good when you're talking about ranges in 5%. But realistically, when m is less than 10, no one is do anything in the 5% range, so it's not that big of a deal just to ignore it.

So what we did is we populated this table just now. So we took a look at what's the villain's range up here and what's our range? So when we push 100% of the time and he calls 30% of the time, we win 39% of the time. Where red means that we are not likely to win and green means that we are likely to win. Like when we push 5% and he calls with anything, we're 73% to win.

So that's what this table is telling us, and we want to do is we want to come up with an equation that lets us populate this table without actually having to do the range versus range calculations by hand. So what I did was I just ran a regression based on logarithmic variables, and I found something that I found to be really, really cool.

So these seem to be the coefficients with an r squared of 98. So this equation seems to basically nail these range versus range equities, which is, I think, really cool.

So our guess is probably a reason that it starts at 50%, and it seems to be symmetrical. Which is-- I don't know if there's a statistical reason for that, but that's extremely fascinating. And it could make sense intuitively that-- so this is your chance of winning. So would it make sense that the wider he calls-- when we take the natural log of the percentage of him calling, our percent win goes up.

So I think it does because it means he's calling with a worse hand. That's why it

goes up when he calls greater, but then goes down when we push greater because this means that we're pushing worse hands. So that's the equation there, and that's giving us something that we can actually differentiate when we're trying to solve this. And just in terms of errors, it's not that bad. We have an r squared of 98.

So I think this is good enough to use. We can actually use this to try to optimize our decision making preflop. So we solved-- we came up with an equation that we can use for determining our chance of winning a hand compared to our pushing range and his calling range.

So let's go back to our EV model for semi-bluff. So we already did that in the fold equity portion, but we're going to build this out to be relevant to our situation in particular. So we're assuming we're talking about in terms of M, so what's our-- so our blinds are going to be equal to 1. Like this means this is 1 M and our EV is going to be in terms of M's. We're going to-- if our EV ends up being 0.35, it means 0.35 times all the blinds combined just to get rid of blinds here.

So all the blinds combined equals M. Our showdown value is just going to be-- so our fold equity is going to be the pot times the chance of him folding, i.e. 1 minus calling. The chance of him-- our showdown value is chance of calling times win amount times win percentage minus lose amount times lose percentage. The win amount is going to be stacked plus 2/3 because if the blinds are 1-2 or something where the big blind is double the small blind, it means that you win his big blind and then you lose your whole stack.

Depending on when we mark our stack, it changes it a little bit with regard to whether we count it as before we pay the blind or after. But it ends up not making a huge difference. It just impacts whether this is plus 2/3 or plus 1/3 or just your stack.

But anyway, we also have this equation, which we just solve for. Where the chance of us winning is related to the chance to him calling and the range that we're pushing. So what you might see here is all of these are related to the same three variables, where it's either the call percentage, the push percentage, or a stack size. So we combine this to one giant equation, which obviously we're not going to

remember but we can use to start solving this mathematically.

So this is what we would end up with. So when M is 1, we end up with this sort of graph. When the hero's push range goes up to 100% here and the villain's call range goes up to 100% here. This is our equity, which I just color-coded so we don't need to look at the numbers. Where green means it's in the hero's favor and yellow means it's close to zero and red means it's in the villain's favor.

So what we see here is it's really green over here and really yellow over there. And this is telling us this is already factoring in fold equity and our chance of winning if he calls. This is saying that if we call 100% of the time, he should call 100% of the time because he wants the yellowest area and we want the greenest area.

So that's with 1 M, so it shouldn't be surprising that our equilibrium at 1 M is going to be everyone like getting all in 100% of the time. We see when we switch to 10 M, it's pretty different. In fact, this 100 is one of the yellowest areas, whereas our green area is either up here or down here.

So we're going to use this to get an idea of how our value changes based on these variables changing to figure out if we can isolate it to a corner and make that our kind of rule. So first, let's find out when we can get a Nash equilibrium.

So the villain gets to pick this. The villain gets to pick his call range and we get to pick our push range. That's the flexibility each person has. So if we push 100% of the time and he calls 100% of the time, this is a Nash equilibrium for 1 M because he can't do any better by going up here because a higher number is worse for him because this is our equity in the hand.

And the equity comes directly from him because it's a heads-up situation whereas we can't do better by pushing left. Like when we go down here, the next number is 0.32, meaning that if we push any less, we're actually losing value because we want the number to be higher. And when it comes to 2 M, 100% isn't that much off. We're talking about 0.01 M of a difference in EV, but we do actually reach a Nash equilibrium here-- 2 M-- where we can't do anything any better by moving and he

can't do any better by moving.

So when M is 3, we have an unstable Nash, and this is when stuff is getting interesting. If we push 80, he should call 100. If he calls 100, we should push 65, and so on. So we end up in a bit of a circle. It's a rock, paper, scissors situation.

What do we have? M equals 3. So this lets us know that this whole Nash equilibrium thing is not going to work out. Like it only really works for M of 2, and that's not the most important situation to figure out because you're know that you should be pushing a very, very wide range when M is 2.

So we need to figure out some sort of pattern. How do these colors move when your M is more than 2? So let's make-- let's go to M equals 5 and make the gradient a little bit steeper. So these are the same numbers except I just made it-- we're talking about very slight changes in EV, where the difference between yellow here and green here is 0.1 M, and this will help us understand where this value is going to come from.

So let's see how this changes over time, because our goal here is-- the villain's goal is to cause this to be in a yellow area, and the hero's goal is to cause this to be in a green area. And you can see why we inherently have this need to figure out what they're going to do. Because say that we're the villain. If we know the hero pushes 100, we call 100. Whereas if we think the hero pushes 30%, we should call 5%.

So let's see how this changes over time. So this is when M is 2, and then this is us increasing as M is 3, 4, 5, 6, 7, 8, 9. We're trying to pick the box that is kind of the yellowest area. Where can we realistically come up with a rule for ending up there? Based on what this x-coordinate is, what should be our y-coordinate? What should be our call? Do we see any way to figure this thing out?

So what I think we should do is just draw a line right here and say let's just figure out what the equation of that line is, and then come up with an estimate of what his pushing range is, and then throw it into whatever function we used to get that line. Which can't be very complicated in order to make sure, no matter what his pushing

range is that we read him for, we end up in this yellow area.

So what I did was for each M, I highlighted what the lowest EV decision is for the hero, meaning the best decision for the villain. And for 10 M, it's this diagonal. But for 1 M, it's all the way down here. Like best you can do is over here. So are our function is we call 10 times his pushing range.

So if we have 1 M and he pushes 5% of the time, we call about 50% of the time. If he pushes 20% of the time, we're calling always, and anything more than 20%, we're always calling. That's where M equals 1. In general, the rule is going to be if M is 1, always call, and if M is 2, you call twice his pushing range.

So if we think he's pushing 50% or more, we're always calling. If we think he's pushing 20%, we call 40% of the time. Like that's the optimal calling range in this heads-up situation.

So when M is 4-- so I skipped one, but when M is 4 we have basically this straight diagonal. We try to match his pushing range. So when he pushes 60, we call 60. When he pushes 100, we call 100. That's the best we can do to dominate his range there.

When M is 6, we call 2/3 his range. And when M is 9, we call half is range. So those are the rules that we're just going to have to remember. When you're in a heads-up situation, if the stack is basically 1 or 2, you're always calling him. At 4, you're calling him an even amount. At 6 and 9, you're calling him like half of what he pushes.

And that's the optimal move. That is how you will-- if you're in a heads-up situation, which you will be at the end of every tournament to the extent that you get there, you will dominate his playing style based on what you read as his pushing amount. And you can just count how many hands did he push versus how many hands does he fold to get an idea of what his range is here.

And this also works when it's folded to you in the small blind. Like even if you're 10 handed, by the time you get to the small blind, you're heads up again and these rules apply. So to the extent you can memorize this thing will give you the right

move in that scenario for Ms up to 10.

So when you're the hero-- when you're the small but, rather, and you're pushing, it requires you to estimate what his calling range is. Because you end up all over the plAce based on making different estimates of what you can call with, and you really don't have any information. But I'm going to graph it out here and then we can see what's going to be a good estimate in basically any scenario.

So there are a couple different ways I can think about this. So we're going to come up with a bunch of numbers based on the villains' call range here. And we can either-- so we're targeting a column and then like the row is going to be information we don't have. So the question is do we pick the column that has the highest average EV, the highest minimum EV, or the highest EV versus like a particular bad player that we're going to be targeting?

And the blue here is what column maximizes your EV for that scenario? And your guess is as good as mine when it comes to what's the best way to strategize it. Are we looking for-- maximizing the min will help us make sure that we're not dominated by someone who's really got our number, whereas maximizing the average might be better when we're trying to figure out against a player we know nothing about what would be better. And maximizing versus tight-- or loose-- will help us figure out some sort of-- how to capitalize on the reads that we're making.

So when M is 1, he's over there. No matter what the scenario, you should be pushing 100% of the time. That's what this is telling us. It shouldn't be a surprise based on all the stuff I told you about 1 M situations. No matter what kind of assumption we're making, push 100%.

So let me pick up to M is 10. So what's going on here? So as M increases, they all kind of move at the same time, except what? If we're talking loose or we're targeting best/worst case scenario or best average, it starts to trickle down to like 50%.

But it's certainly-- they're relatively near each other, and what's good about this is that means that if we target any of these, we're basically in the ballpark where the

difference between this column and this column for any of M is going to be not that material. If you were just in that ballpark, you're fine, but which one of them is completely different.

Yeah, against a tight player. And it should make sense intuitively why if you read him as tight, as someone who only calls 15% of the time, even with 9 and 10 M, you should push 100% of the time.

Why? It's because 85% of the time, he's just going to fold, and even when you're called, the amount of value you get from him folding most of the time just crushes him. So you should-- to the extent that you can encourage him to be tight, do it. But absolutely, if he's tight, push every single hand. You're never in the scenario where pushing less than top 50% is good.

So that's it for heads up. So now let's talk about other positions, and we end up in a lot of complicated situations, which we have to just assume away here. So we can lose in two different ways. We can lose if we call and he beats us, or we could also lose if we call and then someone behind us calls. So we're estimating that someone behind us is only going to call if he has a premium hand because if we're in a short stack situation and someone pushes and then we call, someone is only calling behind us when they have a really good hand.

And let's just assume we're going to lose if we get another caller. Like we are almost certainly going to be dominated-- let's say we have 0 EV there. And because of our equation here, we can actually solve that. We can actually figure out what range is going to be 60% versus another range. So this is resulting in a really cool rule of thumb here, which is when you're in a 10 M or less situation and you're trying to decide whether to call an all-in, you just ask yourself, are you calling such that his range is three times more than your range?

And some questions you might ask yourself is say that you have Ace-10, like you have a 10% range here. Would he push all in into you with King-Jack-- something in a 30%? If you have King-Queen, which is like 30%, would he push all in with 8-5, which is 50% range, and so on.

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So when you're in a [? full ring ?] situation and you're trying to decide whether to call, figure out in general what do you think his pushing range is there. And it's a good call if your calling range is 1/3 of his pushing range. Let's call it a day. Thanks, guys.