Approximations at 0 for $\ln(1+x)$ and $(1+x)^r$

Next, we compute two linear approximations that are slightly more challenging.

Here's the table of values:

$f(x)$	$f'(x)$	$f(0)$	$f'(0)$
$\ln(1+x)$	$\frac{1}{1+x}$	0	1
$(1+x)^r$	$r(1+x)^{r-1}$	1	r

And here are the linear approximations we get from the table:

1. $\ln(1+x) \approx x \quad (\text{if } x \approx 0)$
2. $(1+x)^r \approx 1 + rx \quad (\text{if } x \approx 0)$

Remember that we computed the linear approximation to $\ln x$ at $x_0 = 1$. Since our base point wasn't 0 we couldn't include that here. Because $\ln x \to -\infty$ as $x \to 0$, a linear approximation of $\ln x$ near $x_0 = 0$ is useless to us. Instead we have a linear approximation of the function $\ln(1+x)$ near our default base point $x_0 = 0$, which works out to nearly the same thing as a linear approximation of $\ln x$ near $x_0 = 1$.

Similarly, we found a linear approximation to $(1+x)^r$; not to x^r. For some values of r, x^r is not well behaved when $x = 0$. If we really need an approximation of x^r we can get one by a change of variables.

For example, in a previous example we computed that $\ln u \approx u - 1$ for $u \approx 0$ (we've just replaced x by u.) Now we change variables by setting $u = 1 + x$. If we plug in $1 + x$ everywhere we had a u we get:

$$\ln(1+x) \approx (1+x) - 1 = x,$$

which is exactly the formula we have above.

If you've memorized $\ln(1+x) \approx x$ for $x \approx 0$ you can quickly find an approximation for $\ln u$ for $u \approx 1$ through the change of variables $x = u - 1$.