

**Example:**  $\frac{e^{-3x}}{\sqrt{1+x}}$

Last lecture we computed the linear approximation for  $x$  near 0 of

$$\frac{e^{-3x}}{\sqrt{1+x}} = e^{-3x}(1+x)^{-1/2}.$$

This lecture we'll compute a quadratic approximation for this function when  $x$  is near 0.

To do this we need to use the quadratic approximations for  $e^{-3x}$  and  $(1+x)^{-1/2}$ . We'll use the following two approximation formulas:

$$\begin{aligned}e^x &\approx 1 + x + \frac{1}{2}x^2 \\(1+x)^r &\approx 1 + rx + \frac{r(r-1)}{2}x^2\end{aligned}$$

substituting  $x = 3x$  into the first and  $r = -\frac{1}{2}$  into the second.

$$e^{-3x}(1+x)^{-1/2} \approx \left(1 + (-3x) + \frac{1}{2}(-3x)^2\right) \left(1 + \left(-\frac{1}{2}\right)x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}x^2\right)$$

This looks awful! But we can ignore any terms of higher degree than  $x^2$  and avoid doing those multiplications when we apply the distributive law, so it's not as bad as it looks.

$$e^{-3x}(1+x)^{-1/2} \approx 1 - 3x - \frac{1}{2}x + \frac{3}{2}x^2 + \frac{9}{2}x^2 + \frac{3}{8}x^2$$

Now we combine like terms:

$$e^{-3x}(1+x)^{-1/2} \approx 1 - \frac{7}{2}x + \frac{51}{8}x^2$$

Remember that this approximation is only valid for  $x \approx 0$ , and notice that the first two terms are exactly the linear approximation we got last time.

As you can see, calculations with quadratic approximations are much more involved than those with just linear approximations.

**Question:** Why do we get to drop all the higher order terms?

**Answer:** Because in the situation in which we're going to apply this,  $x$  is a very small number like  $\frac{1}{100}$ . That means that  $x^2 \approx \frac{1}{10000}$  and  $x^3 \approx \frac{1}{1000000}$ . We don't need an exact answer so we can safely ignore anything as small as a millionth, which is what our  $x^3$  terms represent.

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18.01SC Single Variable Calculus  
Fall 2010

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