Relative Error

We continue with our example of time dilation in GPS satellite operation. We started with the following formula from special relativity:

$$T_m = \frac{T}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and used a linear approximation to find that:

$$T_m \approx T\left(1 + \frac{1}{2}\left(\frac{v^2}{c^2}\right)\right).$$

This formula describes the difference due to time dilation between clocks on the ground and on the satellite. Algebraically, the difference is $\Delta T = T_m - T$; it turns out that there's a very simple relation between T, ΔT, v and c:

$$\begin{aligned}
T_m &\approx T\left(1 + \frac{1}{2}\left(\frac{v^2}{c^2}\right)\right) \\
T_m &\approx T + T\frac{1}{2}\left(\frac{v^2}{c^2}\right) \\
T_m - T &\approx T\frac{1}{2}\left(\frac{v^2}{c^2}\right) \\
\Delta T &\approx T\frac{1}{2}\left(\frac{v^2}{c^2}\right) \\
\frac{\Delta T}{T} &\approx \frac{1}{2}\left(\frac{v^2}{c^2}\right)
\end{aligned}$$

In other words, the relative or percent error $\frac{\Delta T}{T}$ caused by time dilation is proportional to the ratio $\frac{v^2}{c^2}$, which relates the speed of the satellite to the speed of light.

As in the example of falling stock prices, this value $\frac{\Delta T}{T}$ gives us an idea of the relative size of the error introduced by time dilation.