Continuity

Continuous Functions

Definition: A function f is *continuous* at x_0 if $\lim_{x \to x_0} f(x) = f(x_0)$.

What is this definition saying? A function that's continuous at x_0 has the following properties:

- $\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x)$; in particular, both of these one sided limits exist.
- $f(x_0)$ is defined.
- $\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = f(x_0)$.

This may look obvious, but remember that when you are calculating $\lim\limits_{x \to x_0} f(x)$ you never allow x to equal x_0. The value $\lim\limits_{x \to x_0} f(x)$ is computed independently of, and in a different way than, the value of $f(x_0)$. If we aren't careful to make this distinction, this definition has no meaning.

The limits for which $\lim\limits_{x \to x_0} f(x) = f(x_0)$ are exactly the "easy limits" we discussed earlier. The "harder" limits only happen for functions that are not continuous.

Next we'll see a tour of different types of discontinuous functions. The question of whether something is continuous or not may seem fussy, but it is something people have worried about a lot. Bob Merton, who was a professor at MIT when he did his work for the Nobel Prize in Economics, was interested in whether stock prices of various kinds are continuous from the left (past) or right (future) in a certain model. That was a serious consideration when developing a model that hedge funds now use all the time.