

## Using The Second Fundamental Theorem of Calculus

This is the quiz question which everybody gets wrong until they practice it.

**Example:** Evaluate  $\frac{d}{dx} \int_1^x \frac{dt}{t^2}$ .

This question challenges your ability to understand what the question means. It looks very complicated, but what it really is is an exercise in recopying!

By definition, we have a function of the form  $G(x) = \int_a^x f(t) dt$  like the one in the second fundamental theorem of calculus. The second fundamental theorem then tells us that  $G'(x) = f(x)$ .

So if  $G(x) = \int_1^x \frac{1}{t^2} dt$ , then  $\frac{d}{dx} G(x) = \frac{1}{x^2}$ .

Despite the fact that this looks like a long, elaborate problem it is actually quite easy to solve.

**Question:** Why don't you integrate?

**Answer:** Because we don't need to! When you take the derivative of something and then the antiderivative, you get back to the original thing (plus some arbitrary constant). In this case, we're taking the antiderivative and then the derivative; again we get back to the same thing. There isn't even an arbitrary constant to worry about here because the derivative of that constant is zero.

Let's see what happens when we integrate. It's slower than just recopying, but we can check it to see that the theorem does work.

$$\int_1^x t^{-2} dt = -t^{-1} \Big|_1^x$$

(Remember that  $t$  is our variable of integration, not  $x$ .)

$$\int_1^x t^{-2} dt = -\frac{1}{x} - (-1).$$

Then we take the derivative to get  $\frac{d}{dx} \int_1^x \frac{dt}{t^2}$ :

$$\frac{d}{dx} \left( 1 - \frac{1}{x} \right) = 0 - (-x^{-2}) = \frac{1}{x^2}.$$

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