

## Summary of Trig Integration

We now know the following facts about trig functions and calculus:

$$\sec x = \frac{1}{\cos x} \quad \tan x = \frac{\sin x}{\cos x} \quad \sin^2 x + \cos^2 x = 1$$

$$\csc x = \frac{1}{\sin x} \quad \cot x = \frac{\cos x}{\sin x} \quad \sec^2 x = 1 + \tan^2 x$$

$$\frac{d}{dx} \tan x = \sec^2 x \quad \frac{d}{dx} \sec x = \sec x \tan x \quad \int \tan x \, dx = -\ln |\cos(x)| + c$$

$$\frac{d}{dx} \sin x = \cos x \quad \frac{d}{dx} \cos x = -\sin x \quad \int \sec x \, dx = \ln(\sec x + \tan x) + c$$

We've also seen several useful integration techniques, including methods for integrating any function of the form  $\sin^n x \cos^m x$ . At this point we have the tools needed to integrate most trigonometric polynomials.

**Example:**  $\int \sec^4 x \, dx$

We can get rid of some factors of  $\sec x$  using the identity  $\sec^2 x = 1 + \tan^2 x$ . This is a particularly good idea because  $\sec^2 x$  is the derivative of  $\tan x$ .

$$\begin{aligned} \int \sec^4 x \, dx &= \int (1 + \tan^2 x) \sec^2 x \, dx \\ &= \int (1 + \tan^2 x) \sec^2 x \, dx. \end{aligned}$$

Using the substitution  $u = \tan x$ ,  $du = \sec^2 x \, dx$ , we get:

$$\begin{aligned} \int \sec^4 x \, dx &= \int (1 + u^2) du \\ &= u + \frac{u^3}{3} + c \\ \int \sec^4 x \, dx &= \tan x + \frac{\tan^3 x}{3} + c. \end{aligned}$$

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