Summary of Trig Integration

We now know the following facts about trig functions and calculus:

$$\sec x = \frac{1}{\cos x} \qquad \tan x = \frac{\sin x}{\cos x} \qquad \sin^2 x + \cos^2 x = 1$$

$$\csc x = \frac{1}{\sin x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \sec^2 x = 1 + \tan^2 x$$

$$\tfrac{d}{dx} \tan x = \sec^2 x \qquad \tfrac{d}{dx} \sec x = \sec x \tan x \qquad \textstyle\int \tan x\, dx = -\ln|\cos(x)| + c$$

$$\tfrac{d}{dx} \sin x = \cos x \qquad \tfrac{d}{dx} \cos x = -\sin x \qquad \textstyle\int \sec x\, dx = \ln(\sec x + \tan x) + c$$

We've also seen several useful integration techniques, including methods for integrating any function of the form $\sin^n x \cos^m x$. At this point we have the tools needed to integrate most trigonometric polynomials.

Example: $\displaystyle\int \sec^4 x\, dx$

We can get rid of some factors of $\sec x$ using the identity $\sec^2 x = 1 + \tan^2 x$. This is a particularly good idea because $\sec^2 x$ is the derivative of $\tan x$.

$$\begin{aligned} \int \sec^4 x\, dx &= \int (1 + \tan^2 x) \sec^2 x\, dx \\ &= \int (1 + \tan^2 x) \sec^2 x\, dx. \end{aligned}$$

Using the substitution $u = \tan x$, $du = \sec^2 x\, dx$, we get:

$$\begin{aligned} \int \sec^4 x\, dx &= \int (1 + u^2) du \\ &= u + \frac{u^3}{3} + c \\ \int \sec^4 x\, dx &= \tan x + \frac{\tan^3 x}{3} + c. \end{aligned}$$