

Example: $\int \sin^2 x \cos^2 x dx$

To integrate $\sin^2 x \cos^2 x$ we once again use the half angle formulas:

$$\begin{aligned}\cos^2 \theta &= \frac{1 + \cos(2\theta)}{2} \\ \sin^2 \theta &= \frac{1 - \cos(2\theta)}{2}\end{aligned}$$

It's a good idea to do your trigonometric and algebraic manipulations of the integrand off to the side on your paper, so that you don't have to continuously copy over (and maybe forget) the integral sign and the dx .

Side work:

$$\begin{aligned}\sin^2 x \cos^2 x &= \left(\frac{1 - \cos(2x)}{2} \right) \left(\frac{1 + \cos(2x)}{2} \right) \\ &= \frac{1 - \cos^2(2x)}{4}\end{aligned}$$

We still have a square, so we're still not in the easy case. But this is an easier "hard" case, especially since we just computed $\int \cos^2 x dx$. We could use that, but instead let's continue to use half angle formulas until we reach an easy case:

$$\begin{aligned}\sin^2 x \cos^2 x &= \frac{1 - \cos^2(2x)}{4} \\ &= \frac{1}{4} - \frac{1 + \cos(4x)}{4 \cdot 2} \\ &= \frac{1}{8} - \frac{\cos(4x)}{8}\end{aligned}$$

Once we've done the side work we substitute back into the original integral to get:

$$\begin{aligned}\int \sin^2 x \cos^2 x dx &= \int \left(\frac{1}{8} - \frac{\cos(4x)}{8} \right) dx \\ &= \frac{x}{8} - \frac{\sin(4x)}{8 \cdot 4} + c\end{aligned}$$

We should now be able to calculate any integral of the form $\int \sin^n x \cos^m x dx$.

Here's an alternate method of doing the side work using the identity

$$\sin(2\theta) = 2 \sin \theta \cos \theta.$$

$$\begin{aligned}
\sin^2 x \cos^2 x &= (\sin x \cos x)^2 \\
&= \left(\frac{1}{2} \sin(2x)\right)^2 \\
&= \frac{1}{4} \sin^2(2x) \\
&= \frac{1}{4} \left(\frac{1 - \cos(4x)}{2}\right) \\
\sin^2 x \cos^2 x &= \frac{1}{8} - \frac{\cos(4x)}{8}
\end{aligned}$$

Using the double angle formula for the sine function reduces the number of factors of $\sin x$ and $\cos x$, but not quite far enough; it leaves us with a factor of $\sin^2(2x)$. Next, the half angle formula for the sine function allows us to reduce this to a constant minus a multiple of the cosine function.

Note that we get the same expression we did before.

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