

## Derivatives of Hyperbolic Sine and Cosine

Hyperbolic sine (pronounced “sinsh”):

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

Hyperbolic cosine (pronounced “cosh”):

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\frac{d}{dx} \sinh(x) = \frac{d}{dx} \left( \frac{e^x - e^{-x}}{2} \right) = \frac{e^x - (-e^{-x})}{2} = \cosh(x)$$

Likewise,

$$\frac{d}{dx} \cosh(x) = \sinh(x)$$

(Note that this is different from  $\frac{d}{dx} \cos(x)$ .)

Important identity:

$$\cosh^2(x) - \sinh^2(x) = 1$$

Proof:

$$\begin{aligned} \cosh^2(x) - \sinh^2(x) &= \left( \frac{e^x + e^{-x}}{2} \right)^2 - \left( \frac{e^x - e^{-x}}{2} \right)^2 \\ \cosh^2(x) - \sinh^2(x) &= \frac{1}{4} (e^{2x} + 2e^x e^{-x} + e^{-2x}) - \frac{1}{4} (e^{2x} - 2 + e^{-2x}) = \frac{1}{4} (2 + 2) = 1 \end{aligned}$$

Why are these functions called “hyperbolic”?

Let  $u = \cosh(x)$  and  $v = \sinh(x)$ , then

$$u^2 - v^2 = 1$$

which is the equation of a hyperbola.

Regular trig functions are “circular” functions. If  $u = \cos(x)$  and  $v = \sin(x)$ , then

$$u^2 + v^2 = 1$$

which is the equation of a circle.

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