Translating $y = 1$ into Polar Coordinates

We'll take a simple description from rectangular coordinates, $y = 1$, and translate it into polar coordinates. To do this, we plug in the (definitive) formula $y = r\sin\theta$.

$$\begin{aligned} y &= r\sin\theta \\ 1 &= r\sin\theta \\ r &= \frac{1}{\sin\theta} \end{aligned}$$

In rectangular coordinates the line has equation $y = 1$. In polar coordinates its equation is $r = \dfrac{1}{\sin\theta}$.

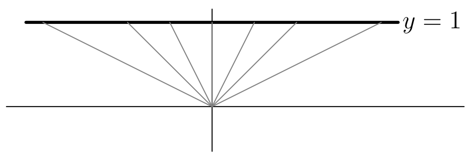

Figure 1: $r = \dfrac{1}{\sin\theta}$

As indicated in Figure 1, for different values of θ points on the horizontal line are different distances r from the origin. That distance r is $\frac{1}{\sin\theta}$.

We need one more piece of information to complete this problem; what is the range of θ? When $\theta = 0$ the denominator of the expression describing r is 0; this corresponds to one end of the line. As θ increases from 0 to π, r decreases to 1 at $\theta = \frac{\pi}{2}$ and then increases to infinity again.

Our final answer is:

$$r = \frac{1}{\sin\theta}, \qquad 0 < \theta < \pi.$$

Question: Is it typical to express r as a function of θ? Does it matter?

Answer: In this course our answers will almost always describe r as a function of θ, but it's not required. We do it this way because we like:

$$r = \frac{1}{\sin\theta}$$

better than:

$$\theta = \sin^{-1}\left(\frac{1}{r}\right).$$

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