Linear Approximation to $\ln x$ at $x = 1$

If you have a curve $y = f(x)$, it is approximately the same as its tangent line $y = f(x_0) + f'(x_0)(x - x_0)$.

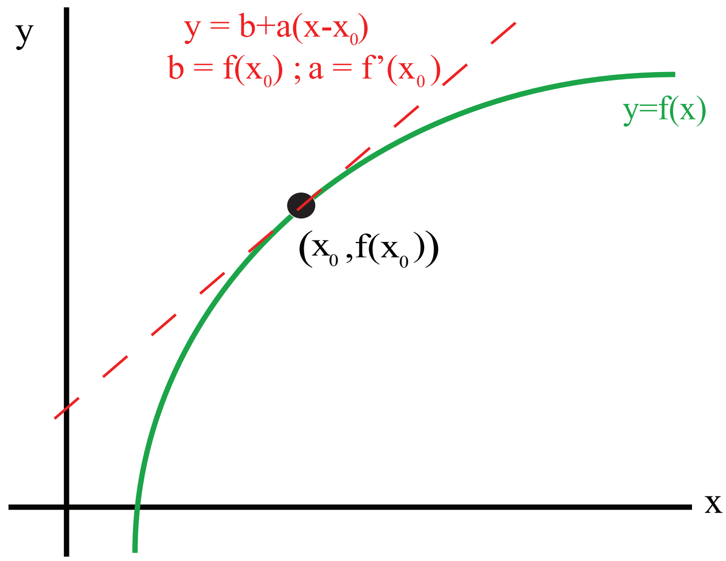

Figure 1: Tangent as a linear approximation to a curve

The tangent line approximates $f(x)$. It gives a good approximation near the tangent point x_0. As you move away from x_0, however, the approximation grows less accurate.

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

Example 1 Let $f(x) = \ln x$. Then $f'(x) = \frac{1}{x}$. We'll use the base point $x_0 = 1$ because we can easily evaluate $\ln 1 = 0$. Note also that $f'(x_0) = \frac{1}{1} = 1$.

Then the formula for linear approximation tells us that:

$$\begin{aligned}
f(x) &\approx f(x_0) + f'(x_0)(x - x_0) \\
\ln x &\approx \ln(1) + 1(x - 1) \\
\ln x &\approx 0 + (x - 1) \\
\ln x &\approx (x - 1)
\end{aligned}$$

Graph the curve $y = \ln x$ and the line $y = x - 1$. You'll see that the two graphs are very close together when $x = x_0 = 1$. You'll also see that they're only near each other when x is near 1.

The point of linear approximation is that the curve (in this case $y = \ln x$) is approximately the same as the tangent line ($y = x - 1$) when x is close to the base point x_0.