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18.02 Multivariable Calculus
Fall 2007

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18.02 Problem Set 12

Due Thursday 12/06/07, 12:45 pm.

Part A (5 points)

Hand in the underlined problems only; the others are for more practice.

- Lecture 34. Thu Nov. 29 Stokes' theorem.
Read: Notes V13; 15.7.
Work: 6F/1ab, 2, 3, 5.
- Lecture 35. Fri Nov. 30 Stokes' theorem continued; review.
- Lecture 36. Tue Dec. 4 **Exam 4** covering lectures 28–35.
- Lecture 37. Thu Dec. 6 Applications.
Read: Notes V14, V15. Work: 6G/1; 6H/1, 2.
- Lecture 38. Fri Dec. 7 Final review.
- Lecture 39. Tue Dec. 11 Final review continued.
- Final exam:** Tuesday Dec. 18, 9:00am – 12:00noon, in Johnson.

Part B (9 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Write the names of all the people you consulted or with whom you collaborated and the resources you used.

Problem 1. (Thursday 11/29, 9 points: 2+2+3+2)

Consider the same tetrahedron as in PS 11 Problem 2, with vertices at $P_0 = (0, 0, 0)$, $P_1 = (1, 0, 1)$, $P_2 = (1, 0, -1)$, and $P_3 = (1, 1, 0)$.

a) Say which orientation (order of vertices) of the boundary curve of each face is compatible with the choice of the normal vector pointing out of the tetrahedron.

b) Compute the work done by the vector field $\vec{F} = yz\hat{j} - y^2\hat{k}$ around the boundary curve of the face $P_0P_1P_3$ directly using line integrals (using the orientation from part (a)).

c) Use Stokes' theorem to compute the work done around each of the four faces (including the one you computed directly in part (b)). Use the orientations from part (a). (Note that you can avoid some calculations by re-using your answers from PS 11.)

d) The sum of the four values you found in part (c) should be zero. Explain this in two different ways:

- (i) geometrically, by considering the various line integrals that are being added together;
- (ii) by using the divergence theorem to compute the flux of $\text{curl } \vec{F}$ out of the tetrahedron.