Problems: Simply Connected Regions

1. Is the paraboloid described by $z = x^2 + y^2$ a simply connected surface? Why or why not?

Answer: Yes. Any closed curve C on the surface of the paraboloid can be shrunk to a point. To help visualize this, imagine taking the part of the paraboloid which contains C, smashing it flat, then shrinking C in the resulting planar surface.
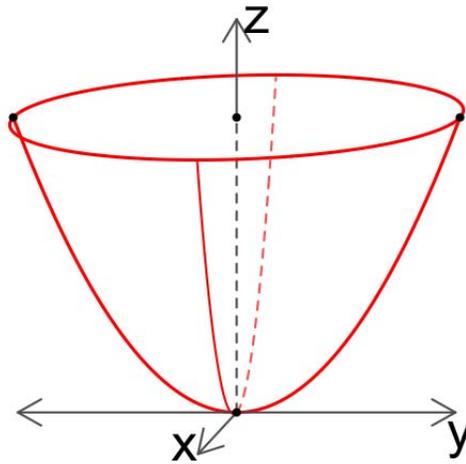
Figure 1: The graph of $z = x^2 + y^2$.

2. Is the Möbius strip described in lecture a simply connected surface? Why or why not?

Answer: No. Think of the Möbius strip as a long, thin rectangle whose ends have been joined to make a loop. A closed curve C around that loop cannot be shrunk to a point without "getting outside of" the Möbius strip.
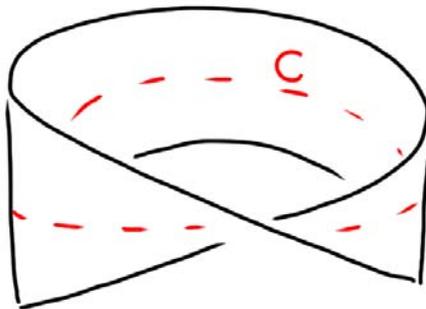
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