Problems: Work and Line Integrals

1. Evaluate $I = \int_C y\,dx + (x+2y)\,dy$ where C is the curve shown.

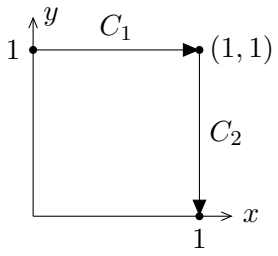


Figure 1: Curve C is C_1 followed by C_2.

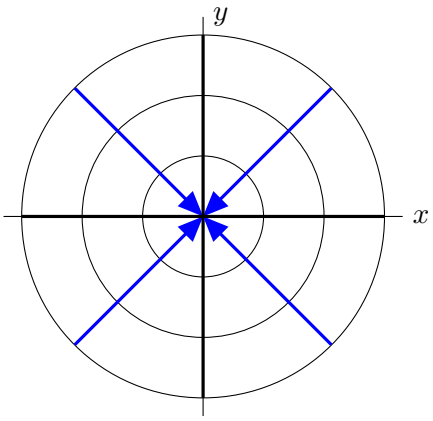
Answer: The curve C is made up of two pieces, so

$$I = \int_{C_1} y\,dx + (x+2y)\,dy + \int_{C_2} y\,dx + (x+2y)\,dy.$$

Note that we don't always need to introduce the variable t.
C_1 : $y = 1$, use x as parameter. $0 \le x \le 1 \;\Rightarrow\; dx = dx,\ dy = 0$.
$\Rightarrow \int_{C_1} y\,dx + (2+2y)\,dy = \int_0^1 dx = 1.$
C_2 : $x = 1$, use y as parameter. y goes from 1 to 0.
$\Rightarrow \int_{C_2} y\,dx + (2+2y)\,dy = \int_1^0 (1+2y)\,dy = -\int_0^1 1+2y\,dy = -2.$
So $I = 1 - 2 = -1$.

2. Let $\mathbf{F} = -x\mathbf{i} - y\mathbf{j}$. Sketch this vector field and describe it in words.

Answer:



Each arrow starts at (x, y) and ends at the origin. The further a vector in this field is from $(0, 0)$, the longer it is.

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