18.02 Exam 2

Problem 1. Let $f(x, y) = x^2y^2 - x$.

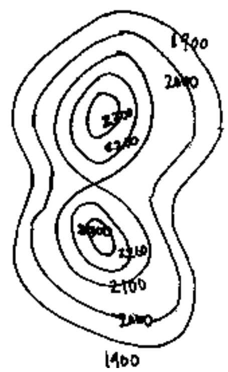
a) (5) Find ∇f at $(2, 1)$

b) (5) Write the equation for the tangent plane to the graph of f at $(2, 1, 2)$.

c) (5) Use a linear approximation to find the approximate value of $f(1.9, 1.1)$.

d) (5) Find the directional derivative of f at $(2, 1)$ in the direction of $-\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

Problem 2. (10) On the contour plot below, mark the portion of the level curve $f = 2000$ on which $\dfrac{\partial f}{\partial y} \geq 0$.



Problem 3. a) (10) Find the critical points of

$$w = -3x^2 - 4xy - y^2 - 12y + 16x$$

and say what type each critical point is.

b) (10) Find the point of the first quadrant $x \geq 0$, $y \geq 0$ at which w is largest. Justify your answer.

Problem 4. Let $u = y/x$, $v = x^2 + y^2$, $w = w(u, v)$.

a) (10) Express the partial derivatives w_x and w_y in terms of w_u and w_v (and x and y).

b) (7) Express $xw_x + yw_y$ in terms of w_u and w_v. Write the coefficients as functions of u and v.

c) (3) Find $xw_x + yw_y$ in case $w = v^5$.

Problem 5. a) (10) Find the Lagrange multiplier equations for the point of the surface

$$x^4 + y^4 + z^4 + xy + yz + zx = 6$$

at which x is largest. (Do not solve.)

b) (5) Given that x is largest at the point (x_0, y_0, z_0), find the equation for the tangent plane to the surface at that point.

Problem 6. Suppose that $x^2 + y^3 - z^4 = 1$ and $z^3 + zx + xy = 3$.

a) (8) Take the total differential of each of these equations.

b) (7) The two surfaces in part (a) intersect in a curve along which y is a function of x. Find dy/dx at $(x, y, z) = (1, 1, 1)$.

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