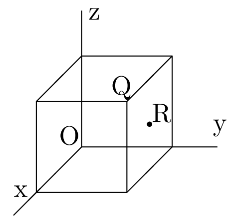
18.02 Practice Exam 1

Problem 1. (15 points)



A unit cube lies in the first octant, with a vertex at the origin (see figure).

a) Express the vectors $\overrightarrow{OQ}$ (a diagonal of the cube) and $\overrightarrow{OR}$ (joining O to the center of a face) in terms of $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$.

b) Find the cosine of the angle between OQ and OR.

Problem 2. (10 points)

The motion of a point P is given by the position vector $\vec{R} = 3\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath} + t\,\hat{k}$. Compute the velocity and the speed of P.

Problem 3. (15 points: 10, 5)

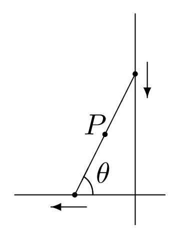
a) Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$; then $\det(A) = 2$ and $A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & a & b \\ -1 & -2 & 5 \\ 2 & 2 & -6 \end{bmatrix}$; find a and b.

b) Solve the system $A\,X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

c) In the matrix A, replace the entry 2 in the upper-right corner by c. Find a value of c for which the resulting matrix M is not invertible.

For this value of c the system $M\,X = 0$ has other solutions than the obvious one $X = 0$: find such a solution by using vector operations. (*Hint:* call U, V and W the three rows of M, and observe that $M\,X = 0$ if and only if X is orthogonal to the vectors U, V and W.)

Problem 4. (15 points)



The top extremity of a ladder of length L rests against a vertical wall, while the bottom is being pulled away. Find parametric equations for the midpoint P of the ladder, using as parameter the angle θ between the ladder and the horizontal ground.

Problem 5. (25 points: 10, 5, 10)

a) Find the area of the space triangle with vertices $P_0 : (2, 1, 0)$, $P_1 : (1, 0, 1)$, $P_2 : (2, -1, 1)$.

b) Find the equation of the plane containing the three points P_0, P_1, P_2.

c) Find the intersection of this plane with the line parallel to the vector $\vec{V} = \langle 1, 1, 1\rangle$ and passing through the point $S : (-1, 0, 0)$.

Problem 6. (20 points: 5, 5, 10)

a) Let $\vec{R} = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$ be the position vector of a path. Give a simple intrinsic formula for $\frac{d}{dt}(\vec{R}\cdot\vec{R})$ in vector notation (not using coordinates).

b) Show that if $\vec{R}$ has constant length, then $\vec{R}$ and $\vec{V}$ are perpendicular.

c) let $\vec{A}$ be the acceleration: still assuming that $\vec{R}$ has constant length, and using vector differentiation, express the quantity $\vec{R}\cdot\vec{A}$ in terms of the velocity vector only.