18.03 Recitation 22, April 29, 2010

Eigenvalues and Eigenvectors

1. We'll solve the system of equations $\begin{cases} \dot{x} &=& -5x - 3y \\ \dot{y} &=& 6x + 4y \end{cases}$

(a) Write down the matrix of coefficients, A, so that we are solving $\dot{\mathbf{u}} = A\mathbf{u}$. What is its trace? Its determinant? Its characteristic polynomial $p_A(\lambda) = \det(A - \lambda I)$? Relate the trace and determinant to the coefficients of $p_A(\lambda)$.

(b) Find the eigenvalues and then for each eigenvalue find a nonzero eigenvector.

(c) Draw the eigenlines and discuss the solutions whose trajectories live on each. Explain why each eigenline is made up o   distinct non-intersecting trajectories. Begin to construct a phase portrait by indicating the direction of time on portions of the eigenlines. Pick a nonzero point on an eigenline and write down all the solutions to $\dot{\mathbf{u}} = A\mathbf{u}$ whose trajectories pass through that point.

(d) Now study the solution $\mathbf{u}(t)$ such that $\mathbf{u}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Write $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as a linear combination of a vector from the first eigenline and a vector from the second eigenline. Use this decomposition to express the solution, and sketch its trajectory. What is the general solution with this trajectory?

(e) Fill out the phase portrait.

2. Same sequence of steps for $\begin{cases} \dot{x} &=& 4x + 3y \\ \dot{y} &=& -6x - 5y \end{cases}$

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