0. Preface

This volume collects, in somewhat organized form, notes I have produced while teaching 18.03, Ordinary Differential Equations, at MIT in 1996, 1999, 2002, and 2004. They are designed to supplement the textbook, which has been Edwards and Penney's *Elementary Differential Equations with Boundary Value Problems*. They are intended to serve several rather different purposes.

In part they try to increase the focus of the course on topics and perspectives which will be found useful by engineering students, while maintaining a level of abstraction, or breadth of perspective, sufficient to bring into play the added value that a mathematical treatment offers.

Here are some specific instances of this.

(1) In this course we use complex numbers, and in particular the complex exponential function, more intensively than Edwards and Penney do, and several of the sections discuss aspects of them.

(2) The "Exponential Response Formula" seems to me to be a linchpin for the course. In the linear theory, when coupled with the complex exponential it leads directly to an understanding of amplitude and phase response curves. (In discussing that, incidentally, I give an introduction to ideas of damping ratio and logarithmic decrement). It has a beautiful extension covering the phenomenon of resonance. It links together the linear theory, the use of Fourier series to study LTI system responses to periodic signals, and the weight function appearing in Laplace transform techniques.

(3) I feel that the standard treatments of Laplace transform in ODE textbooks are wrong to sacrifice the conceptual content of the transformed function, as captured by its pole diagram, and I discuss that topic.

A second purpose is to try to uproot some aspects of standard textbook treatments which I feel are misleading. All textbooks give an account of beats which is artificial and nonsensical from an engineering perspective. As far as I can tell, the derivation of the beat envelope presented here, a simple and revealing use of the complex exponential, is new. Textbooks stress silly applications of the Wronskian, and I try to illustrate what its real utility is. Textbooks tend to make the delta function and convolution seem like part of the theory of the Laplace transform, while I give them independent treatment. Textbooks typically make the theory of first order linear equations seem

quite unrelated to the second order theory; I try to present the first order theory using standard linear methods. Textbooks generally give an inconsistent treatment of the lower limit of integration in the definition of the one-sided Laplace transform, and I try at least to be consistent. As part of this, I offer a treatment (also novel, as far as I can tell) of a class of "generalized functions," which, while artificially restrictive from a mathematical perspective, is sufficient for all engineering applications and which can be understood directly, without recourse to distributions.

A final objective of these notes is to give introductions to a few topics which lie just beyond the limits of this course: the exponential expression of Fourier series; the Gibbs phenomenon; the Wronskian; the initial and final value theorems in the theory of the Laplace transform; the Laplace transform approach to more general systems in mechanical engineering; and an example of linearization, more complicated than the standard pendulum, arising in aeronautical engineering. These essays are not formally part of the curriculum of the course, but they are written from the perspective developed in the course, and I hope that when students encounter them later on, as many will, they will think to look back to see how these topics appear from the 18.03 perspective.

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