# 18.03 Problem Set 6

I encourage collaboration on homework in this course. However, if you do your homework in a group, be sure it works to your advantage rather than against you. Good grades for homework you have not thought through will translate to poor grades on exams. **You must turn in your own writeups of all problems, and, if you do collaborate, you must write on the front of your solution sheet the names of the students you worked with.**

Because the solutions will be available immediately after the problem sets are due, **no extensions will be possible**.

| | | |
|---|---|---|
| L22 | W 31 Mar | Periodic solutions; resonance: EP 8.3, 8.4. |
| R15 | Th 1 Apr | Periodic solutions. |
| L23 | F 2 Apr | Step function and delta function: SN 17. |
| L24 | M 5 Apr | Step response, impulse response: SN 18; Notes IR. |
| R16 | T 6 Apr | Step, delta, and their responses. |
| L25 | W 7 Apr | Convolution: SN 19. |
| R17 | Th 8 Apr | Convolution. |
| L26 | F 9 Apr | Laplace transform: basic properties: EP 4.1. |

**Part I.**

**22. (W 31 Mar)** Notes 7C-1, 7C-2.

**23. (F 2 Apr)** Nothing.

**24. (M 5 Apr)** Find the unit impulse and the unit step response for **(a)** $D + kI$ and **(b)** $D^2 + \omega_n^2 I$.

**25. (W 7 Apr)** Nothing.

**Part II.**

**22. (W 31 Mar)** [Periodic solutions] Let $g(t)$ be the function which is periodic of period $2\pi$, and such that $g(t) = t$ for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ and $g(t) = \pi - t$ for $\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$. (You studied this function in **II.21(f)**.)

**(a)** Find a periodic solution to $\ddot{x} + \omega_n^2 x = g(t)$ (if there is one).

**(b)** For what (positive) values of $\omega_n$ are there no periodic solution?

**(c)** Write $\omega_r$ for the smallest number you found in **(b)**. For $\omega_n$ just less than $\omega_r$, what is the solution like, approximately? How about for $\omega_n$ just larger than $\omega_r$?

**(d)** For what values of $\omega_n$ are there more than one periodic solution?

**(e)** For the values of $\omega_n$ found in **(d)**, are *all* solutions to $\ddot{x} + \omega_n^2 x = g(t)$ periodic?

**23. (F 2 Apr)** [Step and delta] For each of the following functions $f(t)$, (i) draw a graph, (ii) draw a graph of the generalized derivative, (iii) write a formula for $f(t)$ and for $f'(t)$ (with possibly a few values not defined) using $u(t-a)$, $\delta(t-a)$, and other functions.

**(a)** $f(t) = 0$ for $t < 0$, $f(t) = -t$ for $t > 0$.

**(b)** $f(t) = 0$ for $t < 0$, $f(t) = 1 - t$ for $t > 0$.

**(c)** $f(t) = 0$ for $t < 0$, $f(t) = 2t - 1$ for $0 < t < 1$, $f(t) = 0$ for $t > 1$.

**(d)** $f(t) = 0$ for $t < 0$, $f(t) = t - \lfloor t \rfloor$ for $t > 0$, where $\lfloor t \rfloor$ denotes the greatest integer less than or equal to $t$.

**24. (M 5 Apr)** [Step and delta responses]

**(a)** Find the unit impulse response $w$ for the LTI operator $2D^2 + 4D + 4I$.

**(b)** Find the unit step response $v$ for the same operator.

**(c)** Verify that $\dot{v} = w$ (as it should be, since $\dot{u} = \delta$).

**(d)** For each of the following functions, find the LTI differential operator $p(D)$ having it as unit impulse response.

(i) $2u(t)$.

(ii) $u(t)t$.

(iii) $u(t)t^2$.

**25. (W 7 Apr)** [Convolution]

**(a)** Let $q(t) = \cos(\omega t)$. Compute $w(t) * q(t)$ (where $w(t)$ is the unit impulse response for $D + kI$ found in **I.24**) and verify that it is the solution to $\dot{x} + kx = q(t)$ with rest initial conditions.

**(b)** Let $q(t) = 1$. Compute $w(t) * q(t)$ (where $w(t)$ is the unit impulse response for $D^2 + \omega_n^2 I$ found in **I.24**) and verify that it is the solution to $\ddot{x} + \omega_n^2 x = q(t)$ with rest initial conditions.

**(c)** Compute $t^2 * t$ and $t * t^2$. Are they equal?

**(d)** Compute $(t * t) * t$ and $t * (t * t)$. Are they equal?

**I.24 Anwers: For $t > 0$: (a) $w = e^{-kt}$, $v = (1/k)(1 - e^{-kt})$; (b) $w = (1/\omega_n)\sin(\omega_n t)$, $v = (1/\omega_n^2)(1 - \cos(\omega_n t))$.**