18.034 Honors Differential Equations

Spring 2009

18.034 Solutions to Problemset 1

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1. (b) $a = 1$ or $a = 4$

2. (a) $y(\pi/6) = e^2$
   (b) Same as part (a)
   (c) Because $\frac{dx}{x}$ is not integrable on any interval containing the point $x = 0$.

3. (a) y is increasing because $y' = y^2 + 1 > 0$. The formula is obtained by the separation of variables.
   (b) $y(x) = \tan(x - c)$ is defined on the interval $(c - \pi/2, c + \pi/2)$.

4. (b) Let $c = \sup\{x : y(x) = 0\}$.
   If $c = +\infty$, then $y = y_1$. If $c < +\infty$, then $y(x) = 0$ for $x \leq c$ and $y(x) = (x - c)^{3/2}$ for $x > c$ by the separation of variables and uniqueness.

5. Let $q(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$, $a_n \neq 0$
   The general solution $y(x) = c/x + \frac{a_n}{n+1}x^n + \frac{a_{n-1}}{n}x^{n-1} + \ldots + a_0$, $(x \neq 0)$ where c is constant admits the only polynomial solution when $c = 0$.

6. (a) $\frac{1}{1-n}u' + p(x)u = q(x)$.
   (b) $y = u^{-1/2}$ where $u' - 2u = -2x$ for $y \neq 0$.