18.034 Honors Differential Equations
Spring 2009

# 18.034 Practice Midterm #2

Notation. $' = d/dt$.

**1.** (a) Find numbers $a$ and $b$ so that the differential equation $t^2y'' + aty' + by = 0$ has solutions $t^2$ and $t^3$ on the interval $t \in (0, \infty)$.
(b) Find a differential equation that has solutions $(1-t)^2$ and $(1-t)^3$ on the interval $t \in (-\infty, 1)$.
(c) Find a differential equation that has solutions $t$ and $e^t$.

**2.** Using *variation of parameters* find a solution of $y'' - (2/t^2)y = t$, $t \neq 0$.

**3.** Find a general solution of $(D^2-1)^4(D^3+1)^5y = 3e^t$.

**4.** Show that the function $u = e^{\int z}$ is a solution of $y'' + p(t)y' + q(t)y = 0$ if and only if $z$ is a solution of the Riccati equation $y' + p(t)y + q(t) = -y^2$.

**5.** (a) State *the existence and uniqueness theorem* for the initial value problem

$$y' = f(t, y), \qquad y(t_0) = y_0.$$

(b) Show that $f(t, y) = -y + 1$ satisfies the Lipschitz condition for all $t$ and $y$.
(c) Using Picard's iteration method obtain the iterate $y_1(t)$ and $y_2(t)$ of

$$y' = -y + 1, \qquad y(0) = 1.$$

(d) Find the exact solution of the initial value problem in part (c).