18.034 Honors Differential Equations
Spring 2009

18.034 Midterm #1   **Name:**

1. (20 points) Solve the initial value problem
$$y'y'' - t = 0, \qquad y(1) = 2, \quad y'(1) = 1.$$

2. Consider the differential equation $y' = y(5 - y)(y - 4)^2$.
(a) (7 points) Determine the critical points (stationary solutions).

(b) (5 points) Sketch the graph of $f(y) = y(5 - y)(y - 4)^2$.

(c) (8 points) Discuss the stability of critical points in part (b).

3. (20 points) Determine the values of a, if any, for which all solutions of the differential equation
$$y'' - (3-a)y' - 2(a-1)y = 0$$
tend to zero as $t \to \infty$. Here, $' = \frac{d}{dt}$.

4. Consider the undamped forced vibration system

$$y'' + y = 3\cos\omega t, \qquad y(0) = 0, \quad y'(0) = 0.$$

(a) (10 points) Find the solution for $\omega \neq 1$.

(b) (5 points) Find the solution for $\omega = 1$.

(c) (5 points) Discuss the behavior of solutions in part (a) and part (b).

5. (a) (5 points) State the Sturm Comparison Theorem.

(b) (15 points) Show that no nontrivial solution of $y'' + (1 - t^2)y = 0$ vanishes infinitely often on $0 < t < \infty$.