Linear Systems: Introduction

Suppose you have want to model the populations of cats and mice on an island, say $c(t)$ and $m(t)$. Cats feed on mice, and will starve without them. Therefore, any differential equation describing the rate-of-change of c should also involve m. Equally, the rate-of-change of m depends on the current number of cats.

Up until now, we have studied differential equations with a single dependent variable. However, many real-life situations are modeled by collections of differential equations involving several dependent variables and their time-derivatives, the above scenario is one of them.

These are called systems. In this final unit, we will learn about some of the simpler ones: they will be of first order, which means that only the first time-derivatives of the dependents variables are involved. Also, there will usually only be two dependent variables. We will spend most of the unit studying linear systems, before devoting the final few sessions to nonlinear ones.

The present session introduces 2×2 linear systems; we will learn how to solve them by turning the problem into a second order ODE in one variable. Conversely, every second order ODE can be obtained from a system, and such a perspective can shed a new light on the ODE. Look out for the companion matrix and the applet Phase portraits: Matrix entry, which allows one to easily visualize the solutions to these sorts of systems.

Perhaps most importantly, we will learn to present both our system and its solutions using matrices and vectors, paving the way to the later sessions in this unit.

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