Part II Problems

Problem 1: [Sinusoidal input and output]

(a) Express $\text{Re}\left(\dfrac{e^{3it}}{\sqrt{3}+i}\right)$ in the form $a\cos(3t)+b\sin(3t)$. Then rewrite this in the form $A\cos(3t-\phi)$. Now find this same answer using the following method. By finding its modulus and argument, write $\sqrt{3}+i$ in the form $Ae^{i\phi}$. Then substitute this into $e^{3it}/(\sqrt{3}+i)$, and use properties of the exponential function to find B and ϕ such that $\dfrac{e^{3it}}{\sqrt{3}+i} = Be^{i(3t-\phi)}$. Finally, take the real part of this new expression.

(b) Find a solution to the differential equation $\dot{z}+3z = e^{2it}$ of the form we^{2it}, where w is some complex number. What is the general solution?

(c) Find a solution of $\dot{x}+3x = \cos(2t)$ by relating this ODE to the one in **(b)**. What is the general solution?

18.03SC Differential Equations
Fall 2011