Laplace Transform Table

Properties and Rules

Function	**Transform**	
Will learn in this session.		
$f(t)$	$F(s) = \int_{0^-}^{\infty} f(t) e^{-st}\, dt$	(Definition)
$a\, f(t) + b\, g(t)$	$a\, F(s) + b\, G(s)$	(Linearity)
$e^{zt} f(t)$	$F(s-z)$	(s-shift)
Will learn in a future session.		
$f'(t)$	$sF(s) - f(0^-)$	
$f''(t)$	$s^2 F(s) - s f(0^-) - f'(0^-)$	
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0^-) - \quad \cdots - f^{(n-1)}(0^-)$	
$t f(t)$	$-F'(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	
$u(t-a) f(t-a)$	$e^{-as} F(s)$	(t-translation)
$u(t-a) f(t)$	$e^{-as} \mathcal{L}(f(t+a))$	(t-translation)
$(f * g)(t) = \int_{0^-}^{t^+} f(t-\tau)\, g(\tau)\, d\tau$	$F(s)\, G(s)$	
$\int_{0^-}^{t^+} f(\tau)\, d\tau$	$\dfrac{F(s)}{s}$	(integration rule)
Interesting, but not included in this course.		
$\dfrac{f(t)}{t}$	$\int_s^{\infty} F(\sigma)\, d\sigma$	

(The function table is on the next page.)

Function Table

<u>**Function**</u>	<u>**Transform**</u>	**Region of convergence**
Will learn in this session.		
1	$1/s$	$\mathrm{Re}(s) > 0$
e^{at}	$1/(s-a)$	$\mathrm{Re}(s) > a$
t	$1/s^2$	$\mathrm{Re}(s) > 0$
t^n	$n!/s^{n+1}$	$\mathrm{Re}(s) > 0$
$\cos(\omega t)$	$s/(s^2+\omega^2)$	$\mathrm{Re}(s) > 0$
$\sin(\omega t)$	$\omega/(s^2+\omega^2)$	$\mathrm{Re}(s) > 0$
$e^{zt}\cos(\omega t)$	$(s-z)/((s-z)^2+\omega^2)$	$\mathrm{Re}(s) > \mathrm{Re}(z)$
$e^{zt}\sin(\omega t)$	$\omega/((s-z)^2+\omega^2)$	$\mathrm{Re}(s) > \mathrm{Re}(z)$
$\delta(t)$	1	all s
$\delta(t-a)$	e^{-as}	all s
$\cosh(kt) = \dfrac{e^{kt}+e^{-kt}}{2}$	$s/(s^2-k^2)$	$\mathrm{Re}(s) > k$
$\sinh(kt) = \dfrac{e^{kt}-e^{-kt}}{2}$	$k/(s^2-k^2)$	$\mathrm{Re}(s) > k$
Will learn in a future session.		
$\dfrac{1}{2\omega^3}(\sin(\omega t) - \omega t\cos(\omega t))$	$\dfrac{1}{(s^2+\omega^2)^2}$	$\mathrm{Re}(s) > 0$
$\dfrac{t}{2\omega}\sin(\omega t)$	$\dfrac{s}{(s^2+\omega^2)^2}$	$\mathrm{Re}(s) > 0$
$\dfrac{1}{2\omega}(\sin(\omega t) + \omega t\cos(\omega t))$	$\dfrac{s^2}{(s^2+\omega^2)^2}$	$\mathrm{Re}(s) > 0$
$u(t-a)$	e^{-as}/s	$\mathrm{Re}(s) > 0$
$t^n e^{at}$	$n!/(s-a)^{n+1}$	$\mathrm{Re}(s) > a$
Interesting, but not included in this course.		
$\dfrac{1}{\sqrt{\pi t}}$	$\dfrac{1}{\sqrt{s}}$	$\mathrm{Re}(s) > 0$
t^a	$\dfrac{\Gamma(a+1)}{s^{a+1}}$	$\mathrm{Re}(s) > 0$

18.03SC Differential Equations
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