Matrix Methods: Eigenvalues and Normal Modes

In the previous session, we learned how to solve first order systems by elimination; we presented out results using vectors and matrices. This leads to a new approach, pursued in this session; it turns out the system can be largely understood by examining features of the coefficient matrix, notably its eigenvalues and the corresponding eigenvectors.

These are terms belonging to the field of linear algebra; the whole session is fairly algebra-heavy, so we start out by building up some background knowledge. Once this is in place, we develop our approach –first studying a long example, then the general case. It turns out that this splits into subcases classified by features of the eigenvalues of the coefficient matrix: these can be real and distinct; complex; real and repeated. Each case will be accompanied with a worked example.

This session is a quite long, probably *close to two normal sessions in length.* This is because we cover all the possible cases: real, complex and repeated eigenvalues. Please don't rush through it. Take your time and learn this well. Eigenvalues and eigenvectors are not only the key idea in the rest of this unit they are also ubiquitous throughout math, science and engineering.

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