Part I Problems

Find the general solution to the given DE and also the specific solution satisfying the given initial conditions (if any).

Problem 1: $y'' - 3y' + 2y = 0$

Problem 2: $y'' + 2y' - 3y = 0, \qquad y(0) = 1, \ y'(0) = -1.$

In the next three problems, find a DE of the form $ay'' + by' + cy = 0$ which has the given family of solutions $y = c_1y_1 + c_2y_2$, with c_1, c_2 constant.

Problem 3: $y = c_1 + c_2e^{-5x}$

Problem 4: $y = c_1e^{5x} + c_2e^{-5x}$

Problem 5: $y = c_1 + c_2x$

In the next four problems, find the general solution of the given DE.

Problem 6: $y'' - 4y = 0$

Problem 7: $2y'' - 3y' = 0$

Problem 8: $4y'' - 12y' + 9y = 0$

Problem 9: $y^{(4)} - 8y'' + 16y = 0$

Find the general solution to the general DE and also the one satisfying the initial conditions (if any are given).

Problem 10: $y'' + 2y' + 2y = 0$

Problem 11: $y'' - 2y' + 5y = 0; \qquad y(0) = 1, \ y'(0) = -1$

Problem 12: $y'' - 4y' + 4y = 0; \qquad y(0) = 1, \ y'(0) = 1$

Problem 13: Find the general solution to the DE $y'' + 6y' + 9y = 0$

In the next two problems, solve the given initial-value problem.

Problem 14: $y'' - 4y' + 3y = 0, \qquad y(0) = 7, \;\; y'(0) = 11.$

Problem 15: $y'' - 6y' + 25y = 0, \qquad y(0) = 3, \;\; y'(0) = 1$

Problem 16: For the equation $y'' + 2y' + cy = 0, \quad c$ constant,
a) Tell which values of c correspond to each of the three cases: two real roots, repeated real root, and complex roots.
b) For the case of two real roots, tell for which values of c both roots are negative, both roots are positive, or the roots have different signs.
c) Summarize the above information by drawing a c-axis, and marking the intervals on it corresponding to the different possibilities for the roots of the characteristic equation.
d) Finally, use this information to mark the interval on the c-axis for which the corresponding ODE is stable. (The stability criterion using roots is what you will need.)

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