

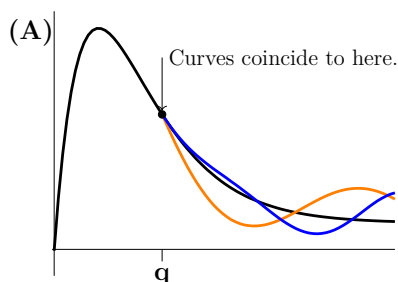
Class 6 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1. Greatest median 1

Each of the curves is the density for a random variable. Where there is just one curve they overlap.

The median of the black plot is at q . Which density has the greatest median?

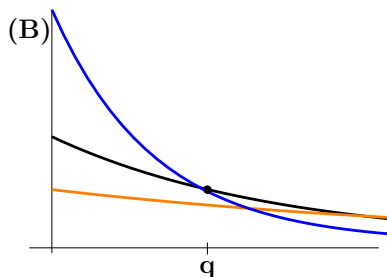


1. Black
2. Orange
3. Blue
4. All the same
5. Impossible to tell

Concept question 2. Greatest median 2

Each of the curves is the density for a random variable. Where there is just one curve they overlap.

The median of the black plot is at q . Which density has the greatest median?



1. Black
2. Orange
3. Blue
4. All the same
5. Impossible to tell

Concept question 3. Desperation

- You have \$100. You need \$1000 by tomorrow morning.
- Your only way to get it is to gamble.
- If you bet \$ k , you either win \$ k with probability p or lose \$ k with probability $1 - p$.

Maximal strategy: Bet as much as you can, up to what you need, each time.

Minimal strategy: Make a small bet, say \$5, each time.

(a) If $p = 0.45$, which is the better strategy?

- (a) Maximal (b) Minimal (c) They are the same
- (b) If $p = 0.8$, which is the better strategy?
- (a) Maximal (b) Minimal (c) They are the same

Board questions

Problem 1.

The random variable X has range $[0,1]$ and pdf $f(x) = cx^2$.

- (a) Find c .
- (b) Find the mean, variance and standard deviation of X .
- (c) Find the median value of X .
- (d) Suppose X_1, \dots, X_{16} are independent identically-distributed copies of X . Let \bar{X} be their average. What is the standard deviation of \bar{X} ?
- (e) Suppose $Y = X^4$. Compute $E[Y]$
- (f) Find the pdf of Y .

Problem 2.

- (a) Make both a frequency and density histogram from the data below.

Use bins of width 0.5 starting at 0. The bins should be right closed.

1	1.2	1.3	1.6	1.6
2.1	2.2	2.6	2.7	3.1
3.2	3.4	3.8	3.9	3.9

- (b) Same question using unequal width bins with edges 0, 1, 3, 4.
- (c) For part (b), why does the density histogram give a more reasonable representation of the data?

Extra problems

Extra 1. Quantiles using R

- (a) Let Z be a standard normal variable ($Z \sim N(0,1)$). Use the R function `qnorm` to find the 0.25, 0.5, 0.75 quantiles for Z .

Use the R help to learn about `qnorm`.

- (b) Graph the pdf of the standard normal distribution. Place the quantiles from part (a) on your graph. Also, indicate on the graph the probabilities connected to the quantiles.

- (c) The R function `pnorm` is the CDF for the normal distribution. In its simplest form `pnorm(z)` is the CDF for the standard normal distribution.

Apply `pnorm` to your quantiles from part (a). Check that the resulting probabilities are exactly what you expect.

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18.05 Introduction to Probability and Statistics

Spring 2022

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