

## 18.06SC Unit 3 Exam

- 1 (34 pts.) (a) If a square matrix  $A$  has all  $n$  of its *singular values* equal to 1 in the SVD, what basic classes of matrices does  $A$  belong to? (Singular, symmetric, orthogonal, positive definite or semidefinite, diagonal)

- (b) Suppose the (orthonormal) columns of  $H$  are eigenvectors of  $B$ :

$$H = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \quad H^{-1} = H^T$$

The eigenvalues of  $B$  are  $\lambda = 0, 1, 2, 3$ . Write  $B$  as the product of 3 specific matrices. Write  $C = (B + I)^{-1}$  as the product of 3 matrices.

- (c) Using the list in question (a), which basic classes of matrices do  $B$  and  $C$  belong to? (Separate question for  $B$  and  $C$ )

- 2 (33 pts.) (a) Find three eigenvalues of  $A$ , and an eigenvector matrix  $S$ :

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

- (b) Explain why  $A^{1001} = A$ . Is  $A^{1000} = I$ ? Find the three diagonal entries of  $e^{At}$ .

- (c) The matrix  $A^T A$  (for the same  $A$ ) is

$$A^T A = \begin{bmatrix} 1 & -2 & -4 \\ -2 & 4 & 8 \\ -4 & 8 & 42 \end{bmatrix}.$$

How many eigenvalues of  $A^T A$  are positive? zero? negative? (Don't compute them but explain your answer.) Does  $A^T A$  have the same eigenvectors as  $A$ ?

**3 (33 pts.)** Suppose the  $n$  by  $n$  matrix  $A$  has  $n$  orthonormal eigenvectors  $q_1, \dots, q_n$  and  $n$  positive eigenvalues  $\lambda_1, \dots, \lambda_n$ . Thus  $Aq_j = \lambda_j q_j$ .

(a) What are the eigenvalues and eigenvectors of  $A^{-1}$ ? *Prove that your answer is correct.*

(b) Any vector  $b$  is a combination of the eigenvectors:

$$b = c_1 q_1 + c_2 q_2 + \cdots + c_n q_n .$$

What is a quick formula for  $c_1$  using orthogonality of the  $q$ 's?

(c) The solution to  $Ax = b$  is also a combination of the eigenvectors:

$$A^{-1}b = d_1 q_1 + d_2 q_2 + \cdots + d_n q_n .$$

What is a quick formula for  $d_1$ ? You can use the  $c$ 's even if you didn't answer part (b).

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