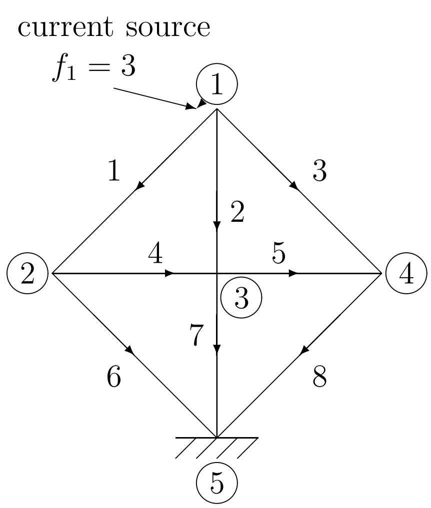
18.085 Quiz 2 November 3, 2004 Professor Strang

Your name is: ______________________________ **Grading** 1.
2.
3.

1) **(36 pts.)** The 5 nodes in the network are at the corners of a *square* and the center. Node 5 is grounded so $x_5 = 0$. All 8 edges have conductances $c = 1$ so $C = I$.



$$A_{8\times 4} = \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

(a) Fill in the 8 by 4 incidence matrix A (node 5 grounded). What is $A^{\mathrm{T}}A$? Is $A^{\mathrm{T}}A$ invertible (YES,NO)?

(b) How many independent solutions to $A^{\mathrm{T}}y = 0$? Write down *one nonzero solution* y.

(c) The current source $f_1 = 3$ enters node 1 and exits at grounded node 5. In 2 by 2 *block form* (using A), what are the 12 equations for the 8 currents y and the 4 potentials x?

(d) Write out *in full with numbers* the 4 equations for the 4 potentials, after the currents y are eliminated. Using symmetry (or guessing or solving) what is the solution x_1, x_2, x_3, x_4?

x

2) **(24 pts.)** The same 8 edges and 5 nodes form a square pin-jointed truss. The pin at node 5 is held in position so $x_5^{\mathrm{H}} = x_5^{\mathrm{V}} = 0$. All 8 elastic constants are $c = 1$ so $C = I$.

(a) How many unknown displacements? ____

What is the shape of the matrix A in $e = Ax$? ____

Find the *first column* of A, corresponding to the stretching e in the 8 edges from a small displacement x_1^{H} at node 1.

(b) Are there any nonzero solutions to $Ax = 0$? (YES,NO)

How many independent solutions do you physically expect? ____

Draw a picture of each independent solution (if any) to show the movement of the 4 nodes.

(c) How many independent solutions to $A^{\mathrm{T}}y = 0$? Can you find them?

xx

3) **(40 pts.)** (a) Find a 4th degree polynomial $s(x, y)$ with only 2 terms that solves Laplace's equation. Please draw a box around your answer $s(x, y)$.

(b) In the xy plane draw all the solutions to $s(x, y) = 0$. Then in the same picture *roughly* draw the curve $s(x, y) = c$ that goes through the particular point $(x, y) = (2, 1)$.

(c) If the curves $s(x, y) = c$ are the *streamlines* of a potential flow (in the usual framework), what is the corresponding velocity $v(x, y) = w(x, y)$?

(d) (this Green's formula question is *not* related to parts a, b, c)

Suppose $w(x, y) = (w_1(x, y), 0)$ is a flow field. With $w_2 = 0$ write down the remaining (not zero) terms in Green's formula for the integral $\iint (\text{grad } u) \cdot w \, dx \, dy$ in the unit square $0 \le x \le 1, 0 \le y \le 1$. Substitute for n and ds when you know what they are for this square.

(e) A one-dimensional formula on any horizontal line $y = y_0$ is integration by parts:

$$\int_{x=0}^{1} \frac{du}{dx} w_1(x) \, dx = - \int_{x=0}^{1} u(x) \frac{dw_1}{dx} \, dx + uw_1(x = 1) - uw_1(x = 0) \, .$$

Here u and w_1 are $u(x, y_0)$ and $w_1(x, y_0)$ since $y = y_0$ is fixed.

Question 1 How do you derive your Green's formula in part (d) from this one-dimensional formula? ANSWER IN ONE SENTENCE, NO MATH SYMBOLS !!

Question 2 (not related) Find all vector fields of this form $(w_1(x, y), 0)$ that can be velocity fields $v = w = (w_1(x, y), 0)$ in potential flow [so $v = \text{grad } u$ and $\text{div } w = 0$ as usual].

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