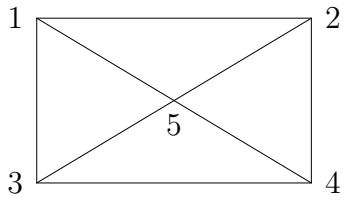
18.085 Computational Science and Engineering I
Fall 2008

18.085 Quiz 2 November 2, 2007 Professor Strang

Your PRINTED name is: _____ **Grading** **1**
2
3

1) **(40 pts.)** This problem is based on a 5-node graph.



I have not included edge numbers and arrows. Add them if you want to: not needed.

(a) Find $A^{\mathrm{T}}A$ for this graph. A is the incidence matrix.

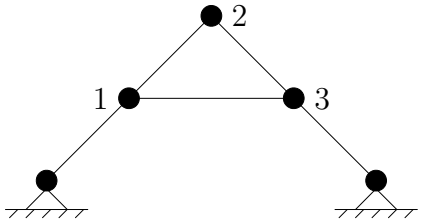
(b) The sum of the eigenvalues of $A^{\mathrm{T}}A$ is _____.
The product of those eigenvalues is _____.

(c) What is $A^{\mathrm{T}}A$ for a graph with **only one edge**? How can that small $A^{\mathrm{T}}A$ be used in constructing $A^{\mathrm{T}}A$ for a large graph?

(d) Suppose I want to solve $Au = \mathrm{ones}(8,1) = b$ by least squares. What equation gives a best $\widehat{u}$? For the incidence matrix A, is there exactly one best $\widehat{u}$ solving that equation? (If your equation has more than one best $\widehat{u}$, describe the difference between any two solutions.)

2) **(30 pts.)** (a) Suppose A is an m by n matrix of rank r (so it has r independent columns). How many independent solutions to $Au = 0$ and $A^{\mathrm{T}}w = 0$?

(b) Draw a full set of mechanisms (solutions to $e = Au = 0$ with no stretching) for this truss with unit length bars and 45° angles.



(c) Suppose a mechanism has $u_1^{\mathrm{H}} = .01$. What are u_1^{V} and u_3^{H} and u_3^{V}? What is the actual new length of the bar between joints 1 and 3?

3) **(30 pts.)** This problem is about the equation

$$-u''(x) + u(x) = 1 \qquad \text{with} \quad u(0) = 0 \quad \text{and} \quad u(1) = 0\,.$$

(a) Multiply by a test function $v(x)$. Find the weak form of the equation, after an integration by parts.

(b) With $h = \Delta x = \frac{1}{3}$ draw the admissible piecewise linear trial functions $\phi_1(x), \ldots, \phi_n(x)$. What is n? With test functions = trial functions, give a *formula* for the entry K_{12} in the finite element equation $KU = F$.

(c) Find all the numbers in K and F.