Exercises given with a numbering are from *Basic Analysis: Introduction to Real Analysis (Vol I)* by J. Lebl.

Reading *The Riemann Integral* lecture notes, Section 6.1

Exercises

1. (a) Suppose that $f \in C([a,b])$, $f(x) \geq 0$ for all $x \in [a,b]$. Prove that if

$$\int_a^b f = 0,$$

then $f(x) = 0$ for all $f \in [a,b]$.

(b) Suppose that $u \in C([a,b])$ is twice continuously differentiable, $V \in C([a,b])$, $V(x) \geq 0$ for all $x \in [a,b]$ and

$$\begin{aligned} -u''(x) + V(x)u(x) &= 0, \quad x \in [a,b], \\ u(a) = u(b) &= 0. \end{aligned}$$

Prove that $u(x) = 0$ for all $x \in [a,b]$. *Hint: What's one of the most useful theorems in analysis mentioned in Lecture 22?*

2. Exercise 5.3.1
3. Exercise 5.3.9
4. Exercise 6.1.2
5. Exercise 6.1.5