A Laguerre Polynomial-Based Bound on the Symbol Error Probability for Adaptive Antennas With Optimum Combining

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***Abstract*—We derive a simple closed-form upper bound on the symbol error probability for coherent detection of M-ary phase-shift keying using antenna arrays with optimum combining, in the presence of multiple uncorrelated equal-power cochannel interferers and thermal noise in a Rayleigh fading environment. The new bound, based on Laguerre polynomials, is valid for an arbitrary number of antenna elements as well as arbitrary number of interferers, and it is proven to be asymptotically tight. Comparisons with Monte Carlo simulation are also provided, showing that our bound is useful in many cases of interest.**

***Index Terms*—Adaptive arrays, antenna diversity, cochannel interference, eigenvalues distribution, optimum combining (OC), Wishart matrices.**

I. Introduction

ADAPTIVE antennas can significantly improve the performance of wireless communication systems by suitably combining the received signals to reduce fading effects and suppress interference. In particular, with optimum combining (OC), the received signals are weighted and combined to maximize the output signal-to-interference-plus-noise power ratio (SINR). This technique provides substantial improvement in performance over maximal ratio combining (MRC), where the received signals are combined to maximize the desired signal-to-noise ratio (SNR) only, when interference is present.

Closed-form expressions for the bit-error probability (BEP) for coherent detection of binary phase-shift keying (PSK) have been derived for the case of a single nonfading interferer with Rayleigh fading of the desired signal in [1] and [2] and with Rayleigh fading of the desired signal and a single interferer in [3]. In [4], two different methods (direct and moment generation function-based approaches), requiring a single integral with finite limits, are used for BEP evaluation with a single Rayleigh faded interferer. With multiple interferers of arbitrary powers, closed-form expressions of the BEP for PSK with OC are not available in the literature: Thus, Monte Carlo simulation has been used to determine the BEP in [2]. Unfortunately, such simulations are computation intensive and not suitable for system studies where the variations of the average powers of the interferer and the desired signals are considered. An upper bound on the BEP was derived in [5], which is useful for such system studies, but the bound is generally not tight (the required powers for a given BEP can be as much as several decibels from the actual values).

An interesting subclass of interferers is the equal-power interferers case, which generally arises in multiple-input multiple-output (MIMO) systems [6]–[8].[1] For this case, BEP expressions have been derived in [9], involving multiple integrals, as well as BEP approximations for binary modulations in [10], partially based on Monte Carlo simulation results, and a simpler but less accurate approximation in [11]. In [12], a closed-form expression is derived but only for the case where the number of interferers is greater than or equal to the number of antennas (but in this case, the gain of OC is limited) without thermal noise. As before, the upper bound of [5] can be used for system studies, but the looseness of the bound can be an issue.

In this letter, starting from an approach similar to that used in [5], we apply some results on the characteristic polynomial of a complex Wishart matrix to derive new simple upper bounds on the symbol error probability (SEP) for coherent detection of M-ary PSK using OC in the presence of multiple equal-power interferers, as well as thermal noise, in a Rayleigh fading environment. We show that these upper bounds are generally very tight (within a few tenths of a decibel), and therefore, in general, significantly better than previous bounds for the case of multiple equal-power interferers.

In Section II, we describe the system model. Performance and upper bounds are derived in Section III, and in Section IV we compare our analytical bounds with Monte Carlo simulations.

II. System Model

We consider coherent demodulation with OC of multiple received signals in a flat fading environment, as in Fig. 1. The fading rate is assumed to be much slower than the symbol rate. Throughout the letter, $(\cdot)^T$ denotes the transposition operator and $(\cdot)^\dagger$ stands for conjugation and transposition. The received signal at the N_A-element array consists of the desired signal, N_I equal-power interfering signals, and thermal noise. After

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[1]For example, when the cochannel interferer is a MIMO user, multiple equal-power interferers are present. Also, when OC is used to separate desired MIMO signals at the receiver, the interfering signals are equal power.

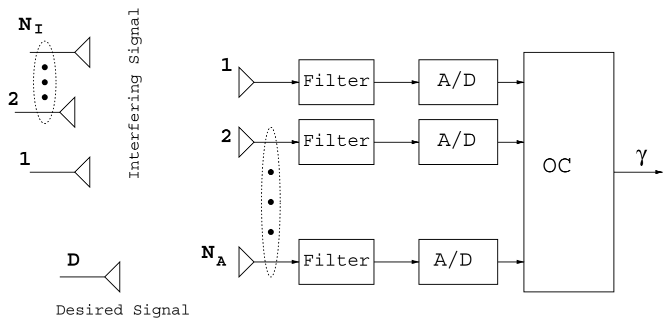

Fig. 1. Baseband model of the OC receiver.

matched filtering and sampling at the symbol rate, the array output vector at time k can be written as

$$\mathbf{z}(k) = \sqrt{E_\mathrm{D}}\boldsymbol{c}_\mathrm{D} b_0(k) + \sqrt{E_\mathrm{I}}\sum_{j=1}^{N_\mathrm{I}} \boldsymbol{c}_{\mathrm{I},j} b_j(k) + \mathbf{n}(k) \tag{1}$$

where E_D and E_I are the mean (over fading) energies of the desired and interfering signals, respectively; $\boldsymbol{c}_\mathrm{D}$ and $\boldsymbol{c}_{\mathrm{I},j}$ are the desired and jth interfering signal propagation vectors, respectively; $b_0(k)$ and $b_j(k)$ (both with unit variance) are the desired and interfering data samples, respectively; and $\mathbf{n}(k)$ represents the additive noise.

The vectors $\boldsymbol{c}_\mathrm{D}$ and $\boldsymbol{c}_{\mathrm{I},j}$ are multivariate complex-valued Gaussian vectors having $\mathbb{E}\{\boldsymbol{c}_\mathrm{D}\} = \mathbb{E}\{\boldsymbol{c}_{\mathrm{I},j}\} = \mathbf{0}$ and $\mathbb{E}\{\boldsymbol{c}_\mathrm{D}\boldsymbol{c}_\mathrm{D}^\dagger\} = \mathbb{E}\{\boldsymbol{c}_{\mathrm{I},j}\boldsymbol{c}_{\mathrm{I},j}^\dagger\} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The jth interfering data samples $b_j(k)$, $j = 1, \ldots, N_\mathrm{I}$ can be modeled as uncorrelated random variables with zero-mean and unit variance.

The additive noise is modeled as a white Gaussian random vector with independent and identically distributed (i.i.d.) elements with $\mathbb{E}\{\mathbf{n}(k)\} = \mathbf{0}$ and $\mathbb{E}\{\mathbf{n}(k)\mathbf{n}^\dagger(k)\} = N_0\mathbf{I}$, where $N_0/2$ is the two-sided thermal noise power spectral density per antenna element.

We also define the SNR as E_D/N_0 and the signal-to-total-interference ratio (SIR) as $E_\mathrm{D}/(N_\mathrm{I}\cdot E_\mathrm{I})$.

The SINR at the output of the N_A-element array with OC can be expressed as [2]

$$\gamma = E_\mathrm{D}\boldsymbol{c}_\mathrm{D}^\dagger \mathbf{R}^{-1}\boldsymbol{c}_\mathrm{D} \tag{2}$$

where the short-term covariance matrix $\mathbf{R}$, conditioned on all interference propagation vectors, is given by

$$\mathbf{R} = E_\mathrm{I}\sum_{j=1}^{N_\mathrm{I}} \boldsymbol{c}_{\mathrm{I},j}\boldsymbol{c}_{\mathrm{I},j}^\dagger + N_0\mathbf{I}. \tag{3}$$

It is important to remark that $\mathbf{R}$, and consequently also the **SINR** γ, vary at the fading rate, which is assumed to be much slower than the symbol rate. Thus, $\mathbf{R}$ is a random matrix, and its eigenvalues are random variables.

The matrix $\mathbf{R}^{-1}$ can be written as $\mathbf{U}\boldsymbol{\Lambda}^{-1}\mathbf{U}^\dagger$, where $\mathbf{U}$ is a unitary matrix and $\boldsymbol{\Lambda}$ is a diagonal matrix whose elements on the principal diagonal are the eigenvalues of $\mathbf{R}$, denoted by $(\lambda_1, \ldots, \lambda_{N_\mathrm{A}})$. The elements of the vector $\mathbf{u} \triangleq \mathbf{U}^\dagger\boldsymbol{c}_\mathrm{D} = [u_1, \ldots, u_{N_\mathrm{A}}]^T$ have the same complex Gaussian distribution as the elements of $\boldsymbol{c}_\mathrm{D}$, since $\mathbf{U}$ represents a unitary transformation. Consequently, the SINR given in (2) can be rewritten as

$$\gamma = E_\mathrm{D}\boldsymbol{c}_\mathrm{D}^\dagger\mathbf{U}\boldsymbol{\Lambda}^{-1}\mathbf{U}^\dagger\boldsymbol{c}_\mathrm{D} = E_\mathrm{D}\sum_{i=1}^{N_\mathrm{A}}\frac{|u_i|^2}{\lambda_i}. \tag{4}$$

By introducing the notation $N_\mathrm{min} = \min\{N_\mathrm{I}, N_\mathrm{A}\}$ and $N_\mathrm{max} = \max\{N_\mathrm{I}, N_\mathrm{A}\}$, it is simple to show that the eigenvalues of the $(N_\mathrm{A}\times N_\mathrm{A})$ matrix $\mathbf{R}$ can be written as

$$\lambda_i = \begin{cases} E_\mathrm{I}\tilde{\lambda}_i + N_0, & i = 1, \ldots, N_\mathrm{min} \\ N_0, & i = N_\mathrm{min}+1, \ldots, N_\mathrm{A} \end{cases} \tag{5}$$

where $\tilde{\lambda}_i$ are the eigenvalues of a complex Wishart matrix,[2] denoted by $\tilde{\mathbf{W}}(N_\mathrm{min}, N_\mathrm{max})$ [9]. Hence, the first N_min eigenvalues λ_i with $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N_\mathrm{min}}$ of $\mathbf{R}$ can be thought of as those of the $(N_\mathrm{min}\times N_\mathrm{min})$ matrix

$$E_\mathrm{I}\tilde{\mathbf{W}}(N_\mathrm{min}, N_\mathrm{max}) + N_0\mathbf{I}. \tag{6}$$

In the next section, we will use the statistical properties of eigenvalues of the complex Wishart matrix to derive a simple upper bound on the SEP with OC.

III. Performance Evaluation

A. Simple Upper Bound on the SEP With OC

The SEP for OC in the presence of multiple cochannel interferers and thermal noise in a fading environment is now obtained by averaging the conditional SEP over the (desired and interfering signals) channel ensemble as $P_e = \mathbb{E}_\gamma\{\Pr\{e|\gamma\}\}$, where $\Pr\{e|\gamma\}$ is the SEP conditioned on the random variable γ. This can be accomplished by using the chain rule of conditional expectation as

$$P_e = \mathbb{E}_{\boldsymbol{\lambda}}\left\{\underbrace{\mathbb{E}_\mathbf{u}\left\{\Pr\{e|\gamma\}\right\}}_{P_{e|\boldsymbol{\lambda}}}\right\} \tag{7}$$

where we first perform $\mathbb{E}_\mathbf{u}\{\cdot\}$ (i.e., average over the channel ensemble of the desired signal) to obtain the conditional SEP, conditioned on the random vector $\boldsymbol{\lambda}$, denoted by $P_{e|\boldsymbol{\lambda}}$. We then perform $\mathbb{E}_{\boldsymbol{\lambda}}\{\cdot\}$ to average out the channel ensemble of the interfering signals.

The Gaussian approximation for cochannel interference contending with the demodulation of the desired signal after minimum-mean-squared-error (MMSE) combining has been shown to give good accuracy in most cases of interest [13]. Thus, owing to the equivalence between OC and MMSE combining [5], the Gaussian approximation will be used hereafter.

Under the assumption of Gaussian interference and noise, $\Pr\{e|\gamma\}$ for coherent detection of M-ary PSK is given (see, for example, [14]–[16]) by

$$\Pr\{e|\gamma\} = \frac{1}{\pi}\int_0^\Theta \exp\left(-\frac{c_\mathrm{MPSK}}{\sin^2\theta}\gamma\right)d\theta \tag{8}$$

[2]Let us define $\mathbf{A}\in M_{m,p}$ with $m \leq p$, where $M_{m,p}$ is the set of the $(m\times p)$ complex matrices, and $\tilde{\mathbf{W}}(m,p) = \mathbf{A}\mathbf{A}^\dagger$. If all the ijth elements of $\mathbf{A}$, a_{ij}, are complex values with real and imaginary parts each belonging to a normal distribution $\mathcal{N}(0, 1/2)$, then the $(m\times m)$ Hermitian matrix $\tilde{\mathbf{W}}(m,p)$ is called Wishart.

where $c_{\text{MPSK}} = \sin^2(\pi/M)$ and $\Theta = \pi(M-1)/M$. Using (8) together with the fact that $\mathbf{u}$ is Gaussian with i.i.d. elements, the conditional SEP $P_{e|\boldsymbol{\lambda}}$, conditioned on $\boldsymbol{\lambda}$, in the general case of N_{A} antennas and N_{I} interferers, becomes

$$\begin{aligned} P_{e|\boldsymbol{\lambda}} &= \frac{1}{\pi}\int_0^{\Theta} \mathbb{E}_{\mathbf{u}}\left\{\exp\left(-\frac{c_{\text{MPSK}}}{\sin^2\theta}E_{\text{D}}\sum_{i=1}^{N_{\text{A}}}\frac{|u_i|^2}{\lambda_i}\right)\right\}d\theta \\ &= \frac{1}{\pi}\int_0^{\Theta}\prod_{i=1}^{N_{\text{A}}}\mathbb{E}_{u_i}\left\{\exp\left(-\frac{c_{\text{MPSK}}}{\sin^2\theta}E_{\text{D}}\frac{|u_i|^2}{\lambda_i}\right)\right\}d\theta \\ &= \frac{1}{\pi}\int_0^{\Theta}\prod_{i=1}^{N_{\text{A}}}\psi\left(-\frac{c_{\text{MPSK}}}{\sin^2\theta}\frac{E_{\text{D}}}{\lambda_i}\right)d\theta \end{aligned} \quad (9)$$

where $\psi(j\nu)$ is the characteristic function of the exponential random variable $|u|^2$ given by

$$\psi(j\nu) = \mathbb{E}\left\{e^{+j\nu|u|^2}\right\} = \frac{1}{1-j\nu}. \quad (10)$$

Therefore

$$P_{e|\boldsymbol{\lambda}} = \frac{1}{\pi}\int_0^{\Theta} A(\theta)\prod_{i=1}^{N_{\min}}\left[\frac{\sin^2\theta}{\sin^2\theta + c_{\text{MPSK}}\frac{E_{\text{D}}}{\lambda_i}}\right]d\theta \quad (11)$$

where

$$A(\theta) \triangleq \left[\frac{\sin^2\theta}{\sin^2\theta + c_{\text{MPSK}}\frac{E_{\text{D}}}{N_0}}\right]^{N_{\text{A}}-N_{\min}}. \quad (12)$$

The derivation of the exact SEP, requiring the evaluation of statistical expectation of (11) with respect of the eigenvalues distribution [9], is not a simple task. In the following theorem, we derive a new upper bound for the SEP based on the expected characteristic polynomial of a complex Wishart matrix.

Theorem 1: The SEP with OC is upper bounded as follows:

$$P_e \le N_{\min}!L_{N_{\min}}^{N_{\max}-N_{\min}}\left(-\frac{N_0}{E_{\text{I}}}\right) \cdot \left(\frac{E_{\text{I}}}{N_0}\right)^{N_{\min}} \cdot P_{e,\text{MRC}}\left(N_{\text{A}},\frac{E_{\text{D}}}{N_0}\right) \quad (13)$$

where

$$P_{e,\text{MRC}}\left(N_{\text{A}},\frac{E_{\text{D}}}{N_0}\right) \triangleq \frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{\sin^2\theta + c_{\text{MPSK}}\frac{E_{\text{D}}}{N_0}}\right]^{N_{\text{A}}}d\theta \quad (14)$$

and $L_n^m(x) = \sum_{k=0}^{n}\binom{n+m}{n-k}((-x)^k/k!)$ are the generalized Laguerre polynomials [17, p. 1061, eq. (8.970)].

Proof: Let us consider the integrand of the conditional SEP of (11) and rewrite

$$\prod_{i=1}^{N_{\min}}\left[\frac{\sin^2\theta}{\sin^2\theta + \frac{c_{\text{MPSK}}E_{\text{D}}}{\lambda_i}}\right] = \frac{\prod_{i=1}^{N_{\min}}\frac{\lambda_i\sin^2\theta}{c_{\text{MPSK}}E_{\text{D}}}}{\prod_{i=1}^{N_{\min}}\left[1+\frac{\lambda_i\sin^2\theta}{c_{\text{MPSK}}E_{\text{D}}}\right]}. \quad (15)$$

Then, by remembering that $\lambda_i = E_{\text{I}}\tilde{\lambda}_i + N_0$, and the fact that E_{I} and $\tilde{\lambda}_i$'s are real and nonnegative, the following inequality holds:

$$\prod_{i=1}^{N_{\min}}\left[1+\frac{\lambda_i\sin^2\theta}{c_{\text{MPSK}}E_{\text{D}}}\right] \ge \prod_{i=1}^{N_{\min}}\left[1+\frac{N_0\sin^2\theta}{c_{\text{MPSK}}E_{\text{D}}}\right]. \quad (16)$$

Therefore, by using (15) and (16), (11) can be upper bounded as follows:

$$P_{e|\boldsymbol{\lambda}} \le \frac{1}{N_0^{N_{\min}}}\left(\prod_{i=1}^{N_{\min}}\lambda_i\right)\frac{1}{\pi}\int_0^{\Theta}\left[\frac{\sin^2\theta}{\sin^2\theta + c_{\text{MPSK}}\frac{E_{\text{D}}}{N_0}}\right]^{N_{\text{A}}}d\theta. \quad (17)$$

Now, in order to apply (7), we need the expectation

$$\mathbb{E}_{\boldsymbol{\lambda}}\left\{\prod_{i=1}^{N_{\min}}\lambda_i\right\} = \mathbb{E}_{\boldsymbol{\lambda}}\left\{\det\left[E_{\text{I}}\tilde{\mathbf{W}}(N_{\min},N_{\max}) + N_0\mathbf{I}\right]\right\}$$

where $\det[\cdot]$ denotes the determinant operator, and the last equality is due to (6) and [18, p. 49, eq. (15)]. Starting from [19, p. 86], it is possible to show, in general, that the expectation of the characteristic polynomial of a complex Wishart matrix $\tilde{\mathbf{W}}(m,p)$ can be written as

$$\mathbb{E}\left\{\det\left[-x\mathbf{I} + \tilde{\mathbf{W}}(m,p)\right]\right\} = L_m^{p-m}(x)m! \quad (18)$$

and therefore

$$\left(\frac{1}{N_0}\right)^{N_{\min}}\mathbb{E}_{\boldsymbol{\lambda}}\left\{\det\left[E_{\text{I}}\tilde{\mathbf{W}}(N_{\min},N_{\max}) + N_0\mathbf{I}\right]\right\} = \left(\frac{E_{\text{I}}}{N_0}\right)^{N_{\min}}N_{\min}!\,L_{N_{\min}}^{N_{\max}-N_{\min}}\left(-\frac{N_0}{E_{\text{I}}}\right). \quad (19)$$

Substituting (17) into (7) and using (19), we obtain (13). This completes the proof of the theorem. □

B. Observations

Comparing (14) with [15, eq. (39)], we see that (14) is the exact expression of the SEP for coherent detection of M-ary PSK using N_{A}-branch MRC in the absence of interference. Note that $P_{e,\text{MRC}}(\cdot,\cdot)$ in (13) is independent of interference, and depends only upon the SNR and the number of antenna elements. Other factors in (13) are independent of SNR, and depend only on the interference-to-noise ratios, the number of interferers, and the number of antenna elements.

It can be observed that (13) is asymptotically tight for $E_{\text{I}} \to 0$: in fact, in this case, as $\lambda_i \to N_0$, the inequality in (16) becomes an equality and the bound tends to the exact solution. It can also be verified that

$$N_{\min}!\,L_{N_{\min}}^{N_{\max}-N_{\min}}(-N_0/E_{\text{I}})\cdot(E_{\text{I}}/N_0)^{N_{\min}}$$

approaches one as $E_{\text{I}}/N_0 \to 0$ (or equivalently as $E_{\text{I}} \to 0$) and hence, as expected, the performance of OC in the absence of interference reduces to that of MRC.

In general, by expanding the Laguerre polynomial, it can be seen that

$$N_{\min}!\,L_{N_{\min}}^{N_{\max}-N_{\min}}(-x^{-1})x^{N_{\min}} = 1 + a_1x + \cdots + a_{N_{\min}}x^{N_{\min}} \quad (20)$$

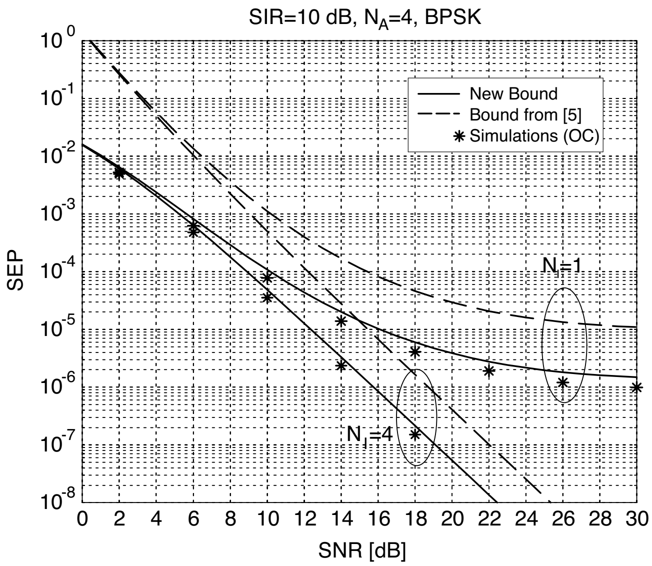

Fig. 2. Comparison of the bounds on SEP for coherent detection of binary PSK using OC with four antennas and SIR $= 10$ dB in the presence of one and four interferers. Also shown in the figure are the results obtained by Monte Carlo simulations.

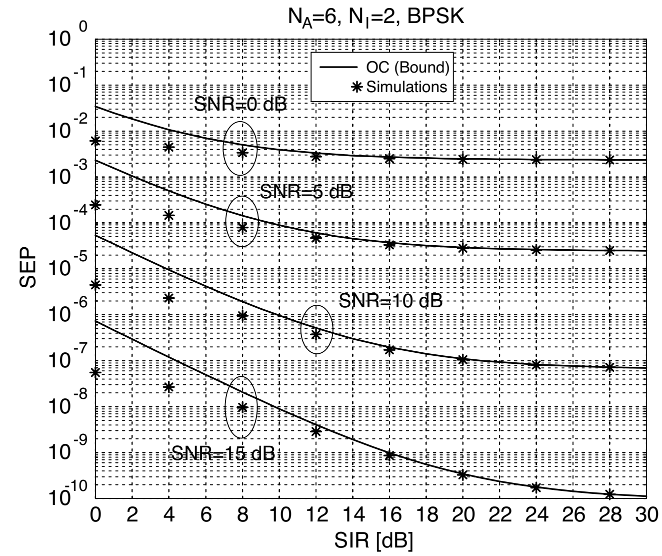

Fig. 3. SEP versus SIR, with SNR as a parameter ranging from 0 to 15 dB; $N_{\rm A} = 6$, $N_{\rm I} = 2$, and BPSK modulation.

is a monic polynomial of degree $N_{\min}$ with nonnegative coefficients in x: It is, therefore, greater than or equal to one. Equation (20) shows that the new bound is in the form of the exact error probability for MRC multiplied by a number greater than or equal to unity; this number, given by (19), represents an upper bound on the increase in SEP due to the presence of interfering signals.

Note also that the definite integral in (14) can be evaluated in closed form using a canonical decomposition method [20], [21]; however, the expression (14) is more compact and clearly displays the dependence of SEP on SNR and diversity order. Moreover, its numerical evaluation is straightforward since the integration has finite limits and the integrand is a simple expression involving trigonometric functions.

Due to the simplicity of the bound, it is now possible, from (13), to obtain many useful results: For instance, the (SNR, SIR)-pairs corresponding to a target SEP (e.g., $\mathrm{SEP} = 10^{-3}$) can be easily derived.

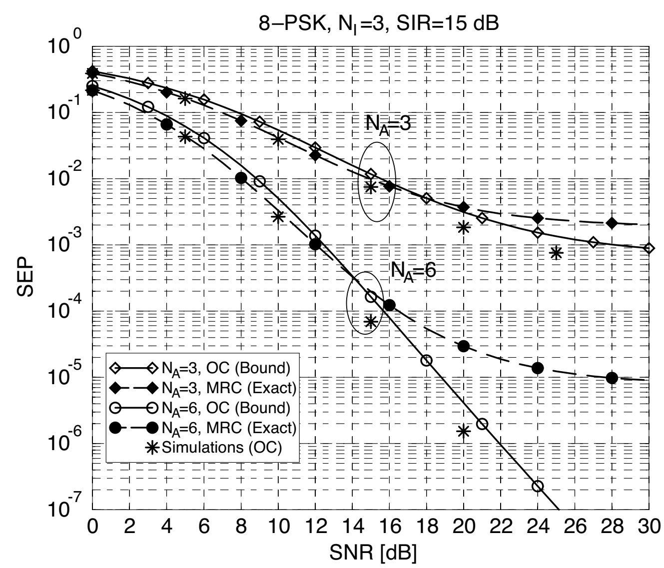

Fig. 4. Comparison between OC and MRC in terms of SEP for coherent detection of 8 PSK with $N_{\rm I} = 3$ and SIR $= 15$ dB. Also shown in the figure are the results obtained by Monte Carlo simulations for OC.

IV. Numerical Results

To assess the validity of the proposed bound, in Fig. 2, we compare our bound (13) and the only previously known bound [5, eq. (13)] on SEP for coherent detection of binary PSK using OC with four antennas and SIR $= 10$ dB in the presence of one and four interferers. Also shown in the figure are the results obtained by Monte Carlo simulations under the same hypothesis leading to (8). It can be observed that the new simple bound is several decibels tighter than the previously known bound, e.g., the difference is more than 4 dB at $\mathrm{SEP} = 10^{-3}$.

Fig. 3 shows the SEP as a function of SIR for several values of SNR, for $N_{\rm A} = 6$, $N_{\rm I} = 2$, and binary phase-shift keying (BPSK) modulation. The comparison with simulation results shows that, for a reasonable range of SIR values, the proposed upper bound is tight for all considered SNR values; a similar behavior (not shown here for the sake of conciseness) can be observed for different values of $N_{\rm I}$ and $N_{\rm A}$.

In Fig. 4, a performance comparison between OC and MRC is presented; the number of interfering signals has been fixed to three, an 8-PSK modulation scheme is considered with SIR $= 15$ dB. Performance with MRC is evaluated by using $P_{e,\mathrm{MRC}}(N_{\rm A}, E_{\rm D}/(N_0 + N_{\rm I} \cdot E_{\rm I}))$, where $P_{e,\mathrm{MRC}}(\cdot,\cdot)$ is given by (14). Also shown in the figure are Monte Carlo simulation results for OC. The results show that, as expected, for small SNR, the thermal noise is dominant and, therefore, MRC and OC perform similarly. On the other hand, for sufficiently large SNR, the role of OC in exploiting the capability of the antenna array is of increasing importance. This aspect is more evident when the number of antennas is larger than that of interferers;

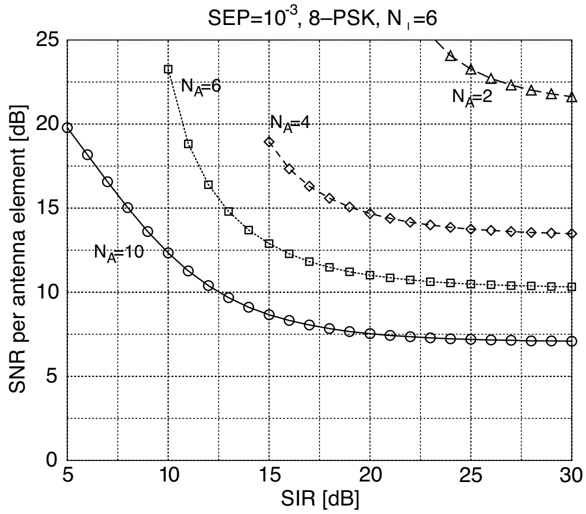

Fig. 5. Bounds on the SNR versus SIR to achieve a target SEP of 10^{-3} for coherent detection of 8 PSK with OC in the presence of six interferers.

e.g., for the case of $N_{\mathrm{A}} = 6$ in Fig. 4, MRC gives rise to an error floor, which is avoided with OC.

Finally, in Fig. 5, for a target SEP $= 10^{-3}$, the locus of points in the (SNR, SIR)-plane has been obtained for $N_{\mathrm{I}} = 6$ and 8 PSK. The curves are for $N_{\mathrm{A}} = 2, 4, 6,$ and 10. Note that the two asymptotes (vertical and horizontal) give the values of SIR and SNR without thermal noise and interference, respectively. The region below each curve represents the outage domain region in which all points produce an SEP higher than target SEP; therefore, if the probability distribution of the SNR and SIR is known, integrating it over the region below each of the curves gives the outage probability corresponding to different numbers of receiving antennas.

Using these results, one can obtain useful information for the design of wireless systems employing OC. For example, for the scenario considered in Fig. 5, if the link budget gives an SNR of 10 dB, a system with $N_{\mathrm{A}} = 10$ requires a minimum SIR of about 12.5 dB to achieve a target SEP $= 10^{-3}$; on the other hand, systems with $N_{\mathrm{A}} < 6$ cannot achieve a target SEP $= 10^{-3}$. Since all results are obtained starting from an upper bound on the SEP, they represent conservative performance estimates from a wireless system designer's point of view.

V. Conclusion

In this letter, we derived, in closed-form, a simple and asymptotically tight upper bound on the SEP for coherent detection of M-ary PSK signals using OC in the presence of multiple equal-power interferers and thermal noise. The new bound, valid for arbitrary numbers of interferers and receiving antennas, has been compared with the only other available bound in the literature as well as Monte Carlo simulations. We also characterized OC in terms of an (SNR, SIR)-plane for a given target SEP, which allows a quick evaluation for the improvement of wireless systems employing OC. Since all results are obtained starting from an upper bound on the SEP, they represent conservative performance estimates from a wireless system designer's point of view.

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