2.001 - MECHANICS AND MATERIALS I
Lecture #21
11/21/2006
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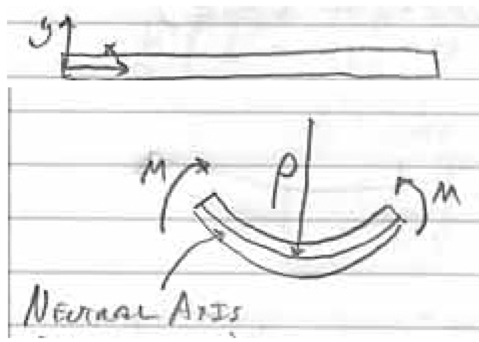
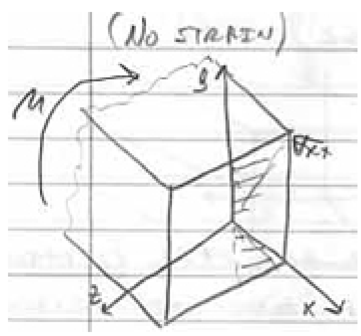
Recall from last time:
Beam Bending



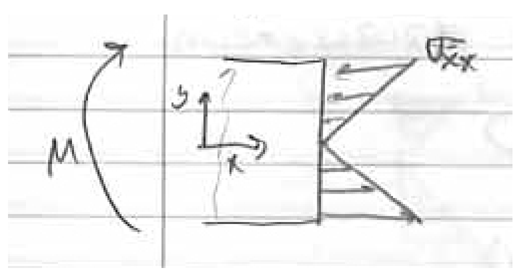
$y = 0$ on neutral axis
$\epsilon_{xx} = \frac{-y}{\rho}$ (Note: purely geometric, no material properties)
$\sigma_{xx} = \epsilon_{xx}E$ (All other σ are equal to 0)

So:

$$\sigma_{xx} = \frac{-Ey}{\rho}$$



Force Equilibrium:



$$\sum F_x = 0$$
$$\int_A \sigma_{xx} dA = 0$$

$$\int_A \frac{Ey}{\rho} dA = 0$$

If E is constant in y then $\int_A ydA - 0$.

Moment Equilibrium

$$\sum M_z = 0$$

$$M = -\int_A \sigma_{xx} y dA$$

$$M = \int_A \frac{Ey^2}{\rho} dA$$

Special case: E constant:

$$M = \frac{1}{\rho} EI$$

$$I = \int_A y^2 dA$$

New this time:

Recall:

$$\sigma_{xx} = \frac{-Ey}{\rho}$$

For constant E (special case):

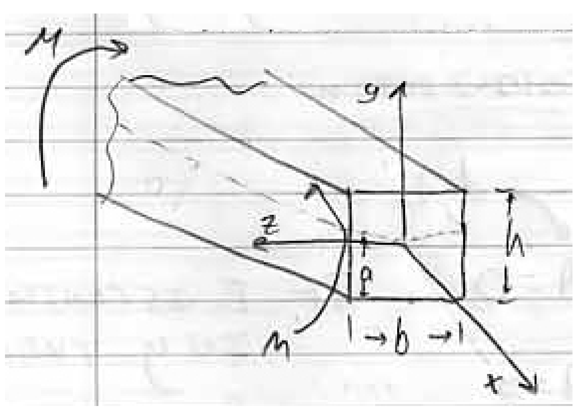
$$M = \frac{EI}{\rho}$$

So:

$$\frac{E}{\rho} = \frac{M}{I} = \frac{-\sigma_{xx}}{y}$$

$$\sigma_{xx} = \frac{-My}{I}$$

EXAMPLE: Find location of neutral axis for rectangular beam

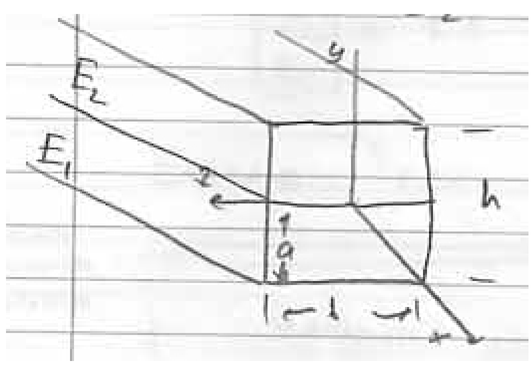


E is constant across cross-section. Recall force equilibrium.

$$\int_A \frac{Ey}{\rho} dA = 0$$
$$\frac{E}{\rho} \int_A y dA = 0$$
$$\frac{E}{\rho} \int_{\frac{-b}{2}}^{\frac{b}{2}} \int_a^{h-a} y dy dz = 0$$
$$\int_{\frac{-b}{2}}^{\frac{b}{2}} \left[\frac{y^2}{2} biggl \right]_a^{h-a} dz = 0$$
$$\frac{1}{2} \int_{\frac{-b}{2}}^{\frac{b}{2}} [(h-a)^2 - a^2] dz = 0$$
$$\frac{1}{2} \int_{\frac{-b}{2}}^{\frac{b}{2}} (h^2 - 2ha) dz = 0$$
$$\frac{1}{2} \left[(h^2 - 2ha) z \right]_{\frac{-b}{2}}^{\frac{b}{2}} = 0$$
$$\frac{1}{2} (h^2 - 2ha)(\frac{b}{2} + \frac{b}{2}) = 0$$
$$\frac{b}{2} (h^2 - 2ha) = 0$$
$$h^2 = 2ha$$
$$a = \frac{h}{2}$$

So the neutral axis is in the center of the beam.

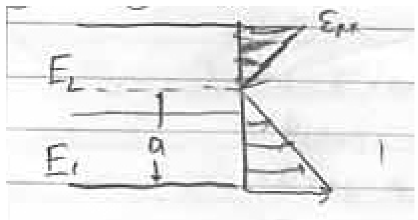
What if $E_2 > E_1$ in:



a is the distance to neutral axis.

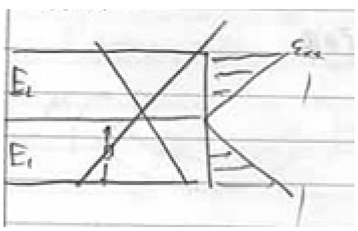
$$\int_A \frac{Ey}{\rho} dA = 0$$

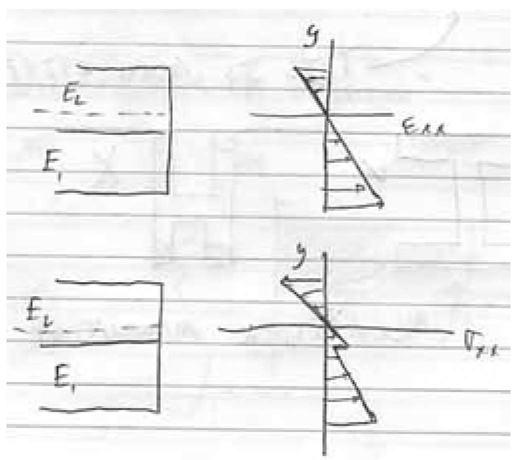
$$\frac{1}{\rho}\int_A E(y) y dA = 0$$



$$\int_{\frac{-b}{2}}^{\frac{b}{2}} \left[\int_{-a}^{-a+\frac{h}{2}} E_1 y dy + \int_{-a+\frac{h}{2}}^{h-a} E_2 y dy \right] dz = 0$$

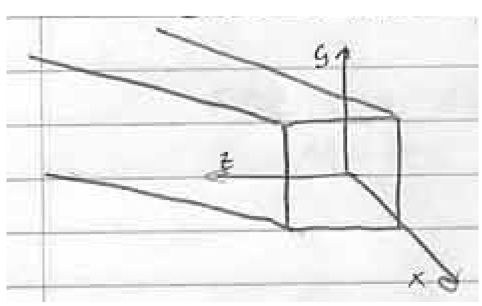
Note:





Example: Moment of Inertia

One material rectangular beam



$$I = \int_A y^2 dA$$

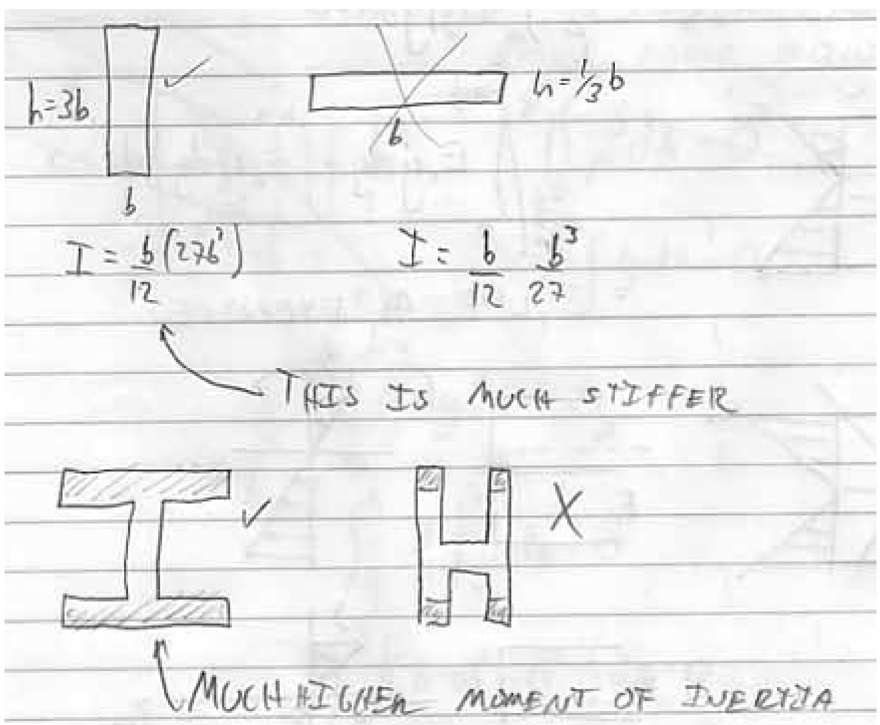
$$I = \int_{\frac{-b}{2}}^{\frac{b}{2}} dz \int_{\frac{-h}{2}}^{\frac{h}{2}} y^2 dy$$

$$I = \int_{\frac{-b}{2}}^{\frac{b}{2}} dz biggl[fracy^3 3\Big]_{\frac{-h}{2}}^{\frac{h}{2}}$$

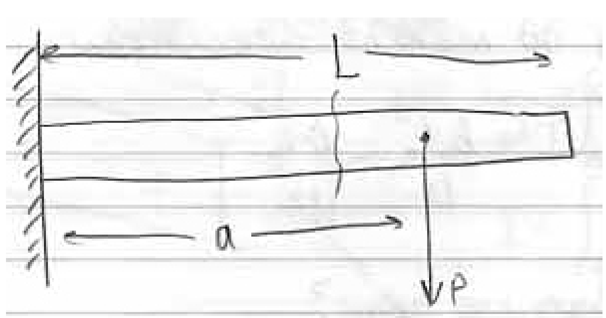
$$I = \int_{\frac{-b}{2}}^{\frac{b}{2}} dz \frac{h^3}{12}$$

$$I = \frac{bh^3}{12}$$

Beam Design

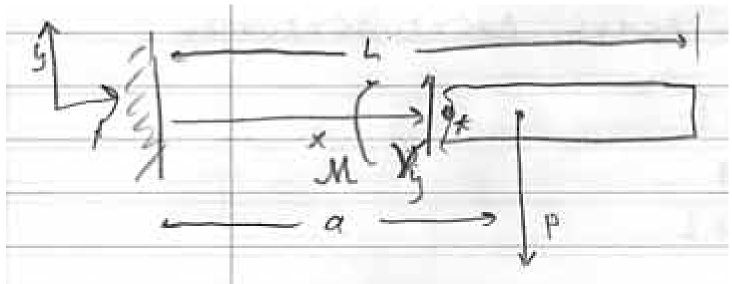


Example:



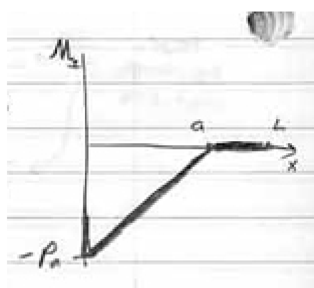
Find σ_{xx}.

FBD:



$$\sum F_y = 0$$
$$V_y - P = 0$$
$$V_y = P$$

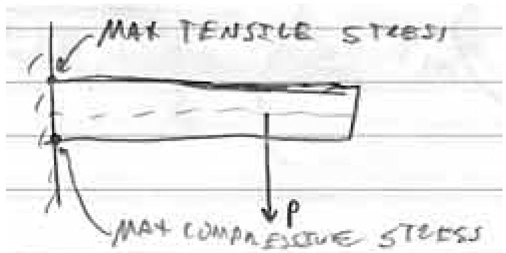
$$\sum M_* = 0$$
$$-M_z - P(a-x) = 0$$
$$M_z = -P(a-x) = 0$$
$$\frac{1}{\rho} = \frac{M_z(x)}{EI}$$



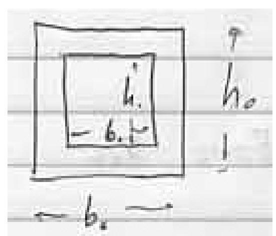
What about shear?

Distortion of planar sections of beam. The can be ignored for slender (long and skinny beams)

$$\sigma_{xx} = \frac{-My}{I} = \frac{-M_z(x)y}{I} = \frac{P(a-x)y}{I}$$



$$\sigma_{xx_max} = \frac{Pa\frac{h}{2}}{\frac{1}{12}bh^3} = \frac{6Pa}{bh^2}$$

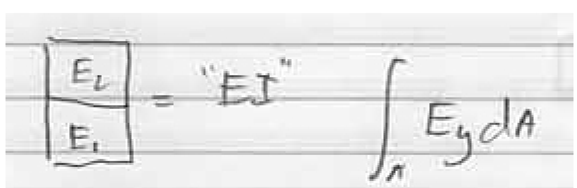


Solve for I

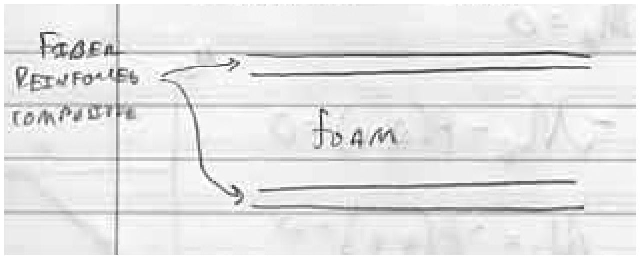
a. Do areas of integration

b. $I = \frac{b_0 h_0^3}{12} - \frac{b_i h_i^3}{12}$

What about a composite beam? This does not work because E was take out of integral during derivation.



Example: Skis



Get good stiffness (bending) but give up axial stiffness and lower weight.

Other examples:
  Plants
  Bird bones
  Airplanes

Recall, x-axis is all for "pure bending"