2.001 - MECHANICS AND MATERIALS I
Lecture #4
9/18/2006
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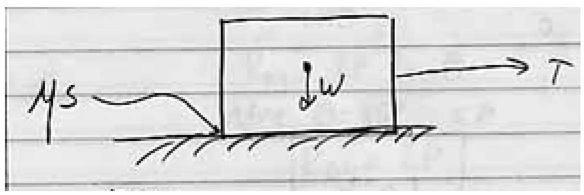
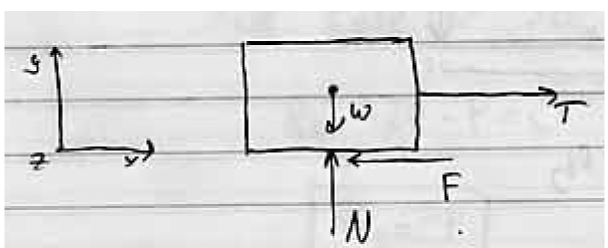
TOPIC: FRICTION

EXAMPLE: Box on floor



μ_s =Coefficient of Static Friction

FBD



Equation of equilibrium

$$\sum F_y = 0$$
$$N - W = 0$$
$$N = W$$

$$\sum F_x = 0$$

$$T - F = 0$$
$$T = F$$

At impending motion *only*:

$$F = \mu_s N$$

For well lubricated, $\mu_s \approx 0.05$.
For very clean surfaces $\mu_s \approx 0.4 - 1$.

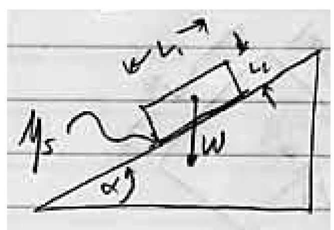
After it starts to move:

$$F = \mu_k N$$

μ_k = Coefficient of kinetic friction.

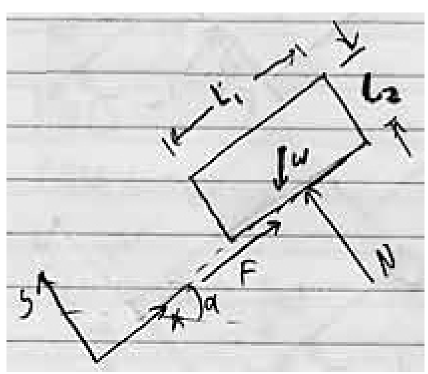
$$\mu_k < \mu_s$$

EXAMPLE: Block on an inclined plane



Q: At what angle (α) does the block slide down the plane?

FBD:



Equilibrium

$$\sum F_x = 0$$
$$F - W \sin\alpha = 0 \Rightarrow F = W \sin\alpha$$

$$\sum F_y = 0$$
$$N - W \cos\alpha = 0 \Rightarrow N = W \cos\alpha$$

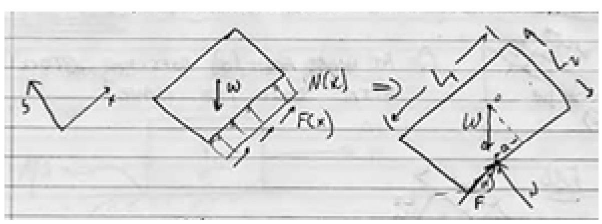
So:

$$\frac{F}{N} = \tan\alpha$$

When you have impending motion:

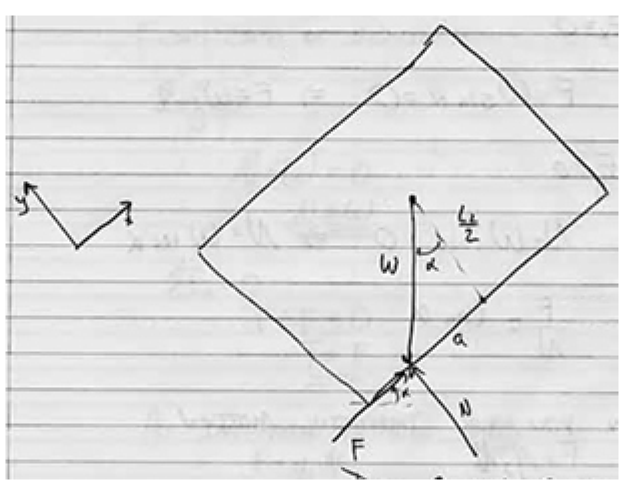
$$F = \mu_s N$$
$$\mu_s = \tan\alpha$$

What about where these forces act?



$$\sum M_0 = 0$$
$$\frac{FL_2}{2} - N_a = 0$$
$$a = \frac{FL_2}{2N} = \frac{L_2}{2}\tan\alpha$$

So:

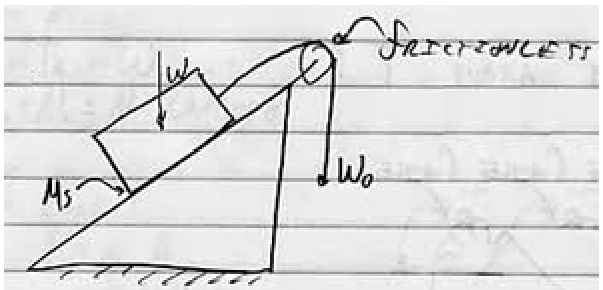


So:

$$a = \frac{L_2}{2} \tan \alpha$$

The resultant of the normal force and frictional force act directly "below" the center of mass.

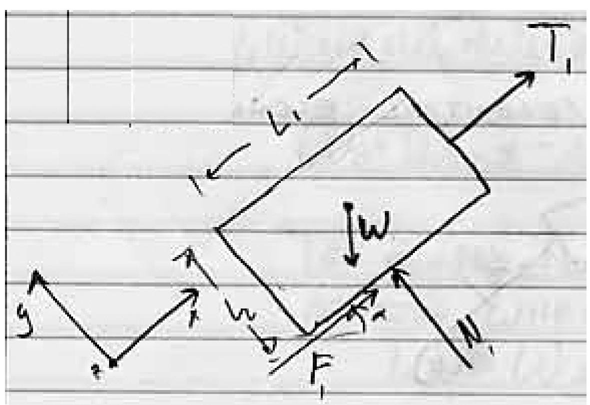
EXAMPLE



Q: For what range of W_0 is the block in equilibrium?

FBD

Case 1: Impending motion is down the plane.



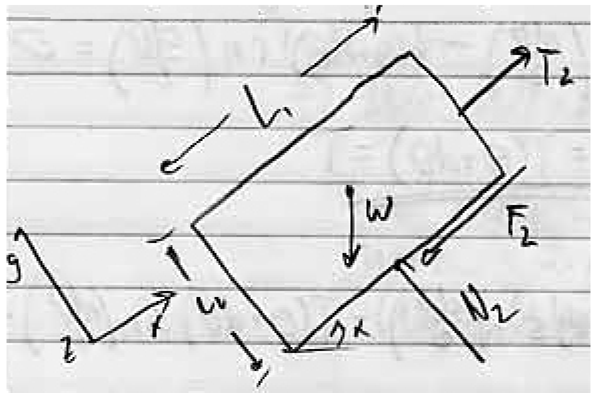
$$\sum F_x = 0$$

$$T_1 + F_1 - W \sin \alpha = 0$$

$$\sum F_y = 0$$

$$N_1 - W \cos \alpha = 0$$

Case 2: Impending motion is up the plane.



$$\sum F_x = 0$$

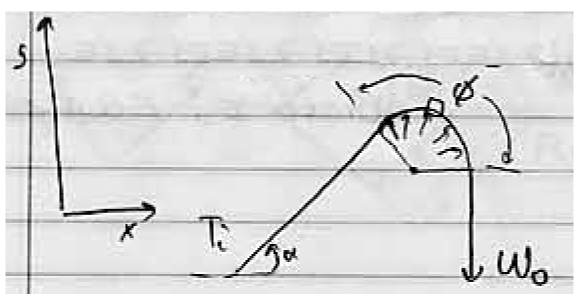
$$T_2 + F_2 - W \sin\alpha = 0$$

$$\sum F_y = 0$$

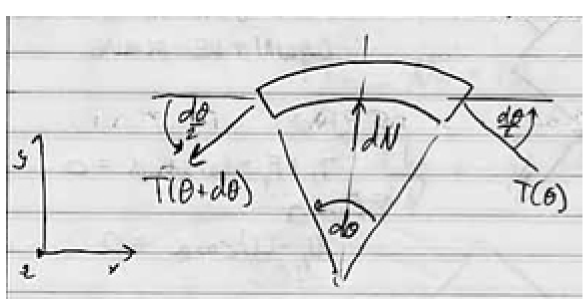
$$N_2 - \cos\alpha = 0$$

What about T?

FBD of Cable



Look at differential element



$$F_x = 0$$
$$T(\theta)\cos\left(\frac{d\theta}{2}\right) - T(\theta + d\theta)\cos\left(\frac{d\theta}{2}\right) = 0$$
$$T(\theta) = T(\theta + d\theta) = T$$

$$F_y = 0$$
$$dN - T(\theta)\sin\left(\frac{d\theta}{2}\right) - T(\theta + d\theta)\sin\left(\frac{d\theta}{2}\right) = 0$$
$$dN - T(\theta)\frac{d\theta}{2} - T(\theta + d\theta)\frac{d\theta}{2} = 0$$
$$Td\theta = dN$$

So:

$$T = W_0$$

Back to block:

$$T_1 = T_2 = W_0$$
$$N_1 = N_2 = N = W\cos\alpha$$

For case 1:

$$F_1 = \mu_s N = \mu_s W\cos\alpha$$

$$\mu_s W\cos\alpha + W_0 + W\sin\alpha = 0$$
$$W_0 = W\sin\alpha - \mu_s W\cos\alpha$$

The block will be stable against downward motion when:

$$W_0 = W\sin\alpha - \mu_s W\cos\alpha$$

For case 2:

$$F_2 = \mu_s N = \mu_s W\cos\alpha$$

$$\mu_s W\cos\alpha + W_0 + W\sin\alpha = 0$$
$$W_0 = W\sin\alpha - \mu_s W\cos\alpha$$

The block will be stable against downward motion when:

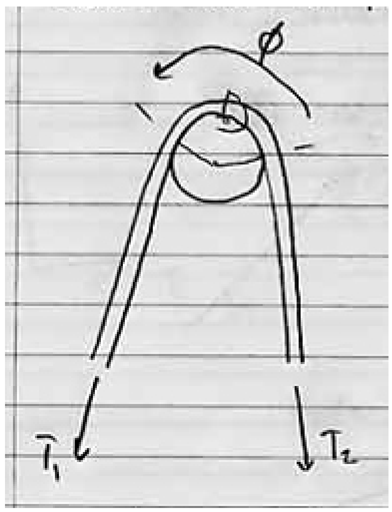
$$W_0 \leq W \sin\alpha + \mu_s W \cos\alpha$$

So it is stable when:

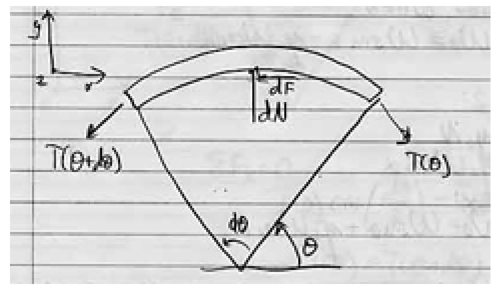
$$W(\sin\alpha - \mu_s \cos\alpha) \leq W_0 \leq W(\sin\alpha + \mu_s \cos\alpha)$$

What about pulley with friction?

Recall a rope around a rod.



Look at a differential element.



$$\sum F_x = 0$$

$$T(\theta)\cos\left(\frac{d\theta}{2}\right) - T(\theta + d\theta)\cos\left(\frac{d\theta}{2}\right) - dF = 0$$

$$\sum F_y = 0$$

$$dN - T(\theta)\sin\left(\frac{d\theta}{2}\right) - T(\theta + d\theta)\sin\left(\frac{d\theta}{2}\right) - dF = 0$$

$$\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$$

$$\cos\left(\frac{d\theta}{2}\right) \approx 1$$

$$dT = T(\theta + d\theta) - T(\theta) \Rightarrow T(\theta + d\theta) = T(\theta) + dT = T + dT$$

So:

$$T(\theta) - T(\theta + d\theta) - dF = 0$$

$$dT = -dF$$

$$dN - \frac{Td\theta}{2} + (T + dT)\frac{d\theta}{2} = 0$$

$$T + dT \to 0$$

$$dN - Td\theta = 0$$

With impending motion:

$$dF = \mu_s dN$$

$$dT = -\mu_s dN$$

$$dN = -\frac{dT}{\mu_s}$$

Substitute:

$$-\frac{dT}{\mu_s} - T d\theta = 0$$

Thus:

$$\frac{dT}{T} = -\mu_s d\theta$$

Integrate:

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\phi} -\mu_s d\theta$$

$$\Big[\ln T\Big]_{T_1}^{T_2} = -\mu_s \phi$$

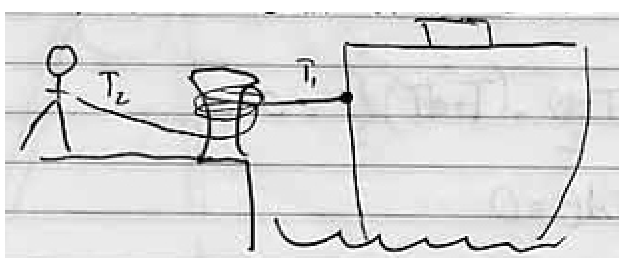
$$\ln\left(\frac{T_2}{T_1}\right) = -\mu_s \phi$$

$$\frac{T_2}{T_1} = \exp(-\mu_s \phi)$$

$$T_2 = T_1 \exp(-\mu_s \phi)$$

This is known as the capstan effect.

EXAMPLE: Boat on a dock



$$\phi = 3(2\pi) \approx 20$$

$$\mu_s = 0.4$$

$$T_2 = T_1 \exp(-8)$$

$$T_2 \approx \left(\frac{1}{3000}\right) T_1$$