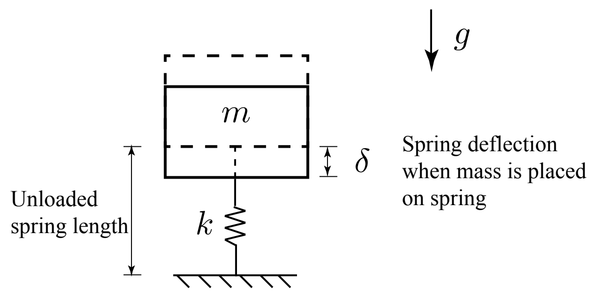
2.003SC
Recitation 10 Notes: Natural Frequency From Deflection & Frequency Response

Obtaining Natural Frequency from Spring Deflection

Consider a spring whose unloaded length is as shown.



When a mass is placed on the spring, in the presence of gravity, the spring deflects due to the mass's weight.

The amount of the deflection can be seen to be

$$\delta = \frac{mg}{k}$$

so

$$k = \frac{mg}{\delta}$$

Recalling that the natural frequency is given by

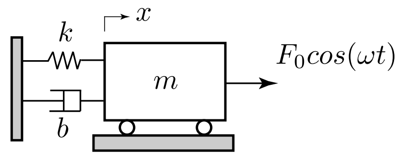
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{\delta m}}$$

or

$$\omega_n = \sqrt{\frac{g}{\delta}}$$

Steady State Frequency Response

Consider a spring-mass-dashpot system subjected to a periodic forcing function.



The equation of motion is

$$m\ddot{x} + b\dot{x} + kx = F_0cos(\omega t)$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}F_0cos(\omega t) = \frac{1}{m}\cdot\frac{k}{k}F_0cos(\omega t)$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + {\omega_n}^2x = \omega_n^2\frac{F_0}{k}cos(\omega t)$$

Let

$$x(t) = \underline{X}_{out}e^{i\omega t} \qquad\qquad \frac{F_0}{k}cos(\omega t) = \underline{X}_{static}e^{i\omega t}$$

Then we can define the transfer function, $\underline{H}(j\omega)$, as the ratio of the output to the input,

$$\underline{H}(j\omega) = \frac{\underline{X}_{out}}{\underline{X}_{static}} = \frac{1}{1 - (\frac{\omega}{\omega_n})^2 + i2\zeta\frac{\omega}{\omega_n}}$$

The transfer function (a complex number) can be resolved into its magnitude and phase. The magnitude and phase are functions of the forcing frequency and are given by

$$|H(\omega)| = \frac{1}{\sqrt{[1-(\frac{\omega}{\omega_n})^2]^2 + [2\zeta(\frac{\omega}{\omega_n})]^2}}$$

and

$$\phi = tan^{-1}\left[\frac{2\zeta(\frac{\omega}{\omega_n})}{1-(\frac{\omega}{\omega_n})^2}\right]$$

Steady State Frequency Response - Problem Statement

A large blower is mounted on a steel frame which is rigidly connected to the floor of a building. The blower's rotor is unbalanced, and subjects the blower to a sinusoidally varying vertical load.

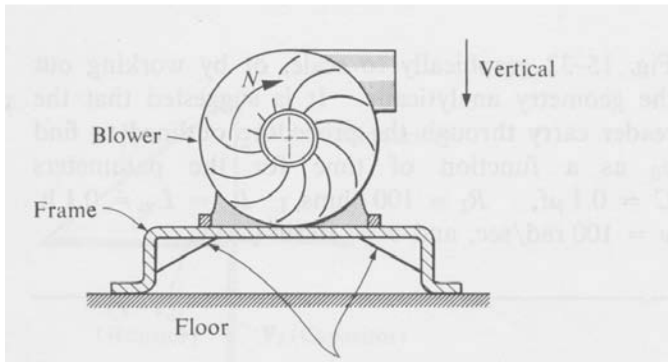


FIGURE 15-35

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The blower weighs 500 lb. When it is placed on the frame, the frame deflects a vertical distance $\delta = 0.026$ in. When the blower is turned on, the blower's rotor spins at $\omega = 1750$ rpm.

When the blower is installed and turned on, it vibrates violently in the vertical direction.

Your assistant has designed some braces which (s)he says will increase the vertical stiffness of the base by about a factor of 2, and believes this will reduce the vibration by about the same factor.

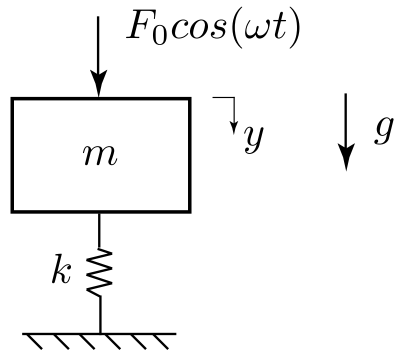
Do you accept the proposal?

Tasks

- Draw a (lumped-parameter) model of the system.
- Determine the system's undamped natural frequency, ω_n , and frequency ratio, $\frac{\omega}{\omega_n}$.
- Estimate the magnitude of the system's frequency response, $|H(\omega)|$.
- Determine the effect of the proposed change.

Steady State Frequency Response - Solution

Model



Frequencies

The forcing frequency is

$\omega = 1750 \left[\frac{rev}{min}\right] \cdot \frac{1}{60} \left[\frac{min}{sec}\right] \cdot 2\pi \left[\frac{radians}{rev}\right] = 183.26 \left[\frac{radians}{sec}\right]$

The deflection in SI units is,

$\delta = 0.026in \cdot \frac{1}{39.37} \left[\frac{meters}{in}\right] = 0.00066$ meters

The undamped natural frequency (before the proposed change) is,

$(\omega_n)^{before} = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.00066}} = 121.88 \left[\frac{radians}{sec}\right]$

The frequency ratio (before the proposed change) is

$(\frac{\omega}{\omega_n})^{before} = \frac{183.26}{121.88} = 1.504$

Magnitude

The magnitude of the frequency response is given by

$$x_{static} \cdot |H(\omega)| = \frac{F_0}{k} \cdot \frac{1}{\sqrt{[1 - (\frac{\omega}{\omega_n})^2]^2 + [2\zeta(\frac{\omega}{\omega_n})]^2}}$$

Evaluating,

$$x_{static} \cdot |H(\omega)|^{before} = \frac{F_0}{k} \cdot (0.8)$$

After the proposed change,

Frequencies

The undamped natural frequency is,

$(\omega_n)^{after} = \sqrt{2}(\omega_n)^{before} = 172.36 \ \ \frac{radians}{sec}$

The frequency ratio is

$(\frac{\omega}{\omega_n})^{after} = \frac{183.26}{172.36} = 1.063$

Magnitude

After the proposed change, the magnitude of the vibration is,

$$x_{static} \cdot |H(\omega)|^{after} = \frac{F_0}{k} \cdot (7.667)$$

Effect of the change...

$$\frac{|H(\omega)|^{after}}{|H(\omega)|^{before}} = \frac{7.667}{0.8} = 9.58$$

The vibration will INCREASE by a factor of about 10 !!! **DON'T MAKE THE CHANGE!!!**

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2.003SC / 1.053J Engineering Dynamics
Fall 2011

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