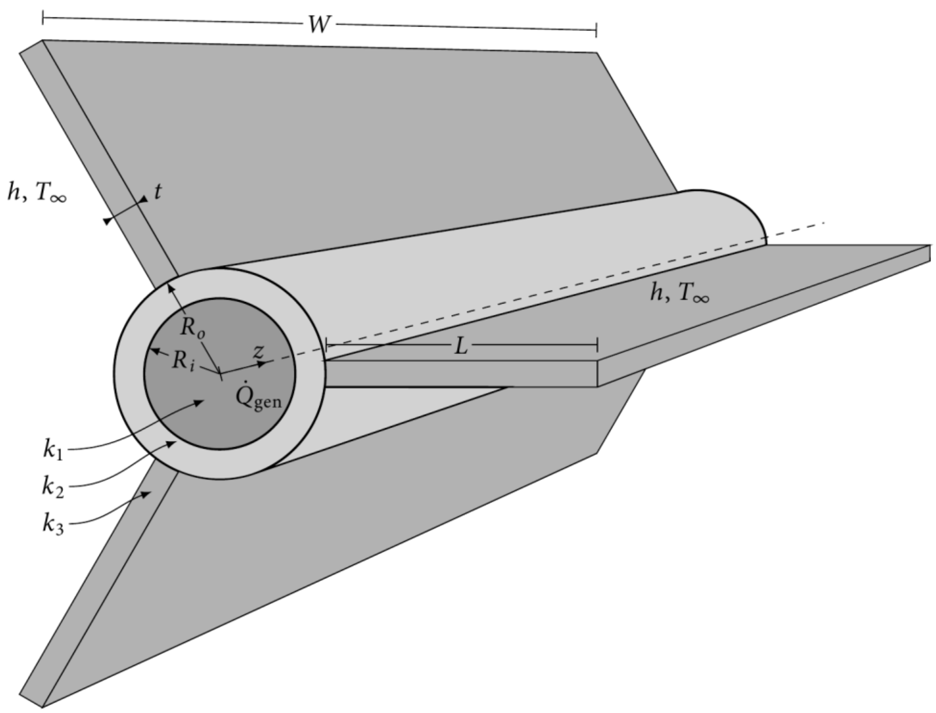
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
2.051 Introduction to Heat Transfer, Fall Term 2015

Quiz 2 Review Problems

Problem 1 (43 points)

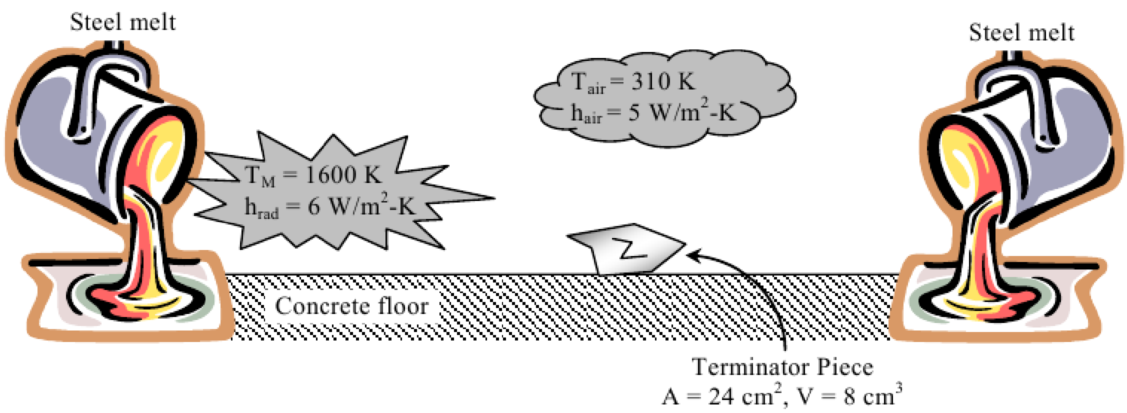
The figure below illustrates a proposed mechanism to provide cooling for a nuclear fuel rod. The fuel rod, which has a radius R_i and thermal conductivity k_1, is surrounded by a *highly conductive*, metallic casing (cladding) whose outer radius is R_o and whose thermal conductivity is k_2. Attached to the cladding are three slender, metallic sheets of thickness t and length L. The thermal conductivity of these sheets is k_3. The device is immersed in a water bath maintained at a constant temperature T_∞. The heat transfer coefficient between the water and the fuel rod device is h. The fuel rod has a length W. Both ends of the device at $z = 0$ and $z = W$ are perfectly insulated. The *total* heat generation rate is $\dot{Q}_{\text{gen}}$ Watts. For the following questions, assume $k_3/(hL) = 100$, $t/L = 0.02$, and the system is at steady state. Express your answer in terms of the quantities illustrated in the figure.

a) (5 points) Describe all relevant modes of heat transfer between from the center of the fuel rod to the surrounding fluid.
b) (3 points) What is the total heat transfer to the water?
c) (14 points) Construct a thermal circuit that models the heat transfer from the inner surface of the cladding ($r = R_i$) to the surrounding fluid. Identify all the resistance elements and write down their expressions in terms of the parameters of the problem
d) (7 points) What is the temperature, $T(R_o)$, at the outer surface of the cladding?
e) (6 points) What is the temperature, $T(R_i)$, at the boundary between the fuel rod and the cladding?
f) (8 points) If the specific heat generation rate in the fuel rod is $\dot{q}_{\text{gen}}$ [W/m^3], and the thermal diffusivity and specific heat of the fuel rod are α and c, what is the temperature $T(0)$, at the center of the fuel rod?



Problem 2: Terminator (25 points)

In the science fiction *classic Terminator 2 – Judgment Day*, the Evil Terminator is made of a malleable metal ($\rho = 2700$ kg/m^3, $c = 900$ J/kg-K, and $k = 240$ W/m-K), which can recombine when molten, thus allowing the Terminator to recover from any mutilation or impact with bullets. In the final scene, the Terminator is shattered into small pieces after being exposed to liquid nitrogen. It seems over, but unfortunately these small pieces land on the concrete floor next to a glowing-hot steel melt. The situation is illustrated in the figure below.



You are to analyze the heat transfer between the Terminator pieces and the air and steel melt. You may assume the following:

- The Terminator pieces have irregular shape, with a surface area $A = 24$ cm^2 and volume $V = 8$ cm^3.
- The initial temperature of the Terminator pieces is equal to the temperature of liquid nitrogen ($T_i = 77$ K).
- The air surrounding the pieces is at $T_{air} = 310$ K and its convective heat transfer coefficient is $h_{air} = 5$ W/m^2-K.
- The steel melt is at $T_M = 1600$ K and the effective heat transfer coefficient for radiation from the steel melt to the pieces is $h_{rad} = 6$ W/m^2-K.
- Neglect heat conduction between the pieces and the floor, as concrete has a very low thermal conductivity.

i) **(5 Points)** Calculate two Biot numbers for the Terminator pieces. One for convection with air and one for radiation with the melt.

ii) **(3 Points)** Determine the direction of the heat transfer between the Terminator pieces and the air at (a) t=0, and (b) the time the Terminator pieces have reached their melting point (800 K).

iii) **(12 Points)** Calculate the time it takes for the Terminator pieces to reach their melting point. Hint: the general solution of the differential equation $\frac{dT}{dt} = -\frac{T}{\tau} + \gamma$ is $T(t) = c_1 e^{-t/\tau} + \gamma t$, where c_1 is a constant of integration.

iv) **(5 Points)** Calculate the entropy transfer rate for the Terminator pieces at $t = 0$. Also, what is the entropy generation rate for the Terminator pieces at $t = 0$?

Problem 3

Asphalt pavement may achieve temperatures as high as 50ºC on a hot summer day. Assume that such a temperature exists throughout the pavement, when suddenly a rainstorm reduces the surface temperature to 20ºC. Calculate the total amount of energy (J/m^2) that will be transferred from the asphalt over a 30-min period in which the surface is maintained at 20ºC.

Problem 4

A tile-iron consists of a massive plate maintained at 150ºC by an embedded electrical heater. The iron is placed in contact with a tile to soften the adhesive, allowing the tile to be easily lifted from the subflooring. The adhesive will soften sufficiently if heated above 50ºC for at least 2 min, but its temperature should not exceed 120ºC to avoid deteriorate of the adhesive. Assume the tile and subfloor to have an initial temperature of 25ºC and to have equivalent thermophysical properties of $k = 0.15$ W/m•K and $\rho c_p = 1.5 \times 10^6$ J/m^2•K.

(a) How long will it take a worker using the tile-iron to lift a tile? Will the adhesive temperature exceed 120ºC?
(b) If the tile-iron has a square surface area 254 mm to the side, how much energy has been removed from it during the time it has taken to lift the tile?

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