MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

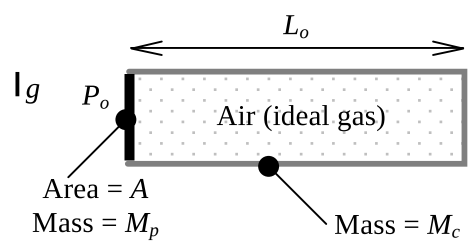
2.06 Fluid Dynamics

RECITATION #3, Spring Term 2013

Topics: Hydrostatics + Surface Tension Examples

Problem 1

Consider the frictionless piston-cylinder system to the right. The mass of the piston and cylinder are M_p and M_c, the cross-sectional area of the piston is A, the length of the cylinder is L_o, and the piston and cylinder walls are very thin. The cylinder is filled with air (modeled as an ideal gas with negligible weight); when the cylinder is held horizontally in ambient pressure P_o as shown, the air occupies its entire volume (i.e., AL_o). Gravitational acceleration is g. The temperature of air may be assumed to be constant throughout the problem.



a) The cylinder is submerged vertically in water (density ρ) and attains equilibrium at depth h as shown in part a) of the figure below. The air and surrounding water are in thermal equilibrium at a temperature T (constant with depth). The goal of part a) is to obtain an expression for the depth h at which the piston-cylinder system is in equilibrium.

   i. By considering a force balance on the entire piston-cylinder-air system, determine the length of the trapped air column L.

   ii. Determine the pressure P of the trapped air.

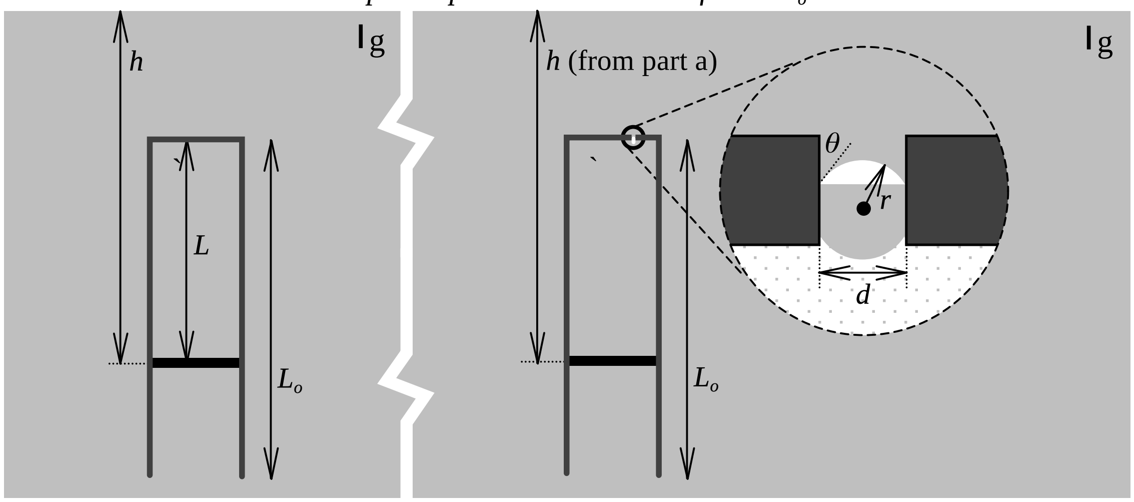
   iii. By considering a force balance on the piston, find the piston depth h as shown.

b) The cylinder now has a small opening of diameter d as shown in part b) of the figure. A meniscus will form at the liquid-air interface; the goal of part b) is to obtain the maximum diameter d of the opening for which the air-water interface is stable *within* the cylindrical hole as shown. Reminder: $\Delta P = \sigma \cdot (1/r_1 + 1/r_2)$, where ΔP is the pressure difference across the interface and r_1 and r_2 are the radii of curvature of the interface.

   i. If the three phase contact angle is θ and the liquid-air surface tension is σ, what is the radius of curvature r of the liquid-air interface and the pressure difference ΔP across the interface, both in terms of given variables?

   ii. By comparing this ΔP to the difference in pressure between the liquid immediately above the cylinder and the trapped air (from part a), find the maximum diameter d for which the air-water interface is stable within the cylindrical opening as shown.

Atmospheric pressure at water surface = P_o



Part a) Part b)