This is the second take-home quiz: Q2

Make a short video (not exceeding 5 minutes), in which using the following slides (the same I used in class) you explain (like I did in class) how we derive the train of inequalities in the last slide.

Before you make your recording, please prepare your tools in advance, in particular set your device or smartphone so as to record with the smallest resolution, possibly 640x480, so that the size is about 25MB/min.

If you do not find a way to make your image appear all the time during the video next to the viewgraphs, please face the camera at least at the beginning so as to identify yourself.

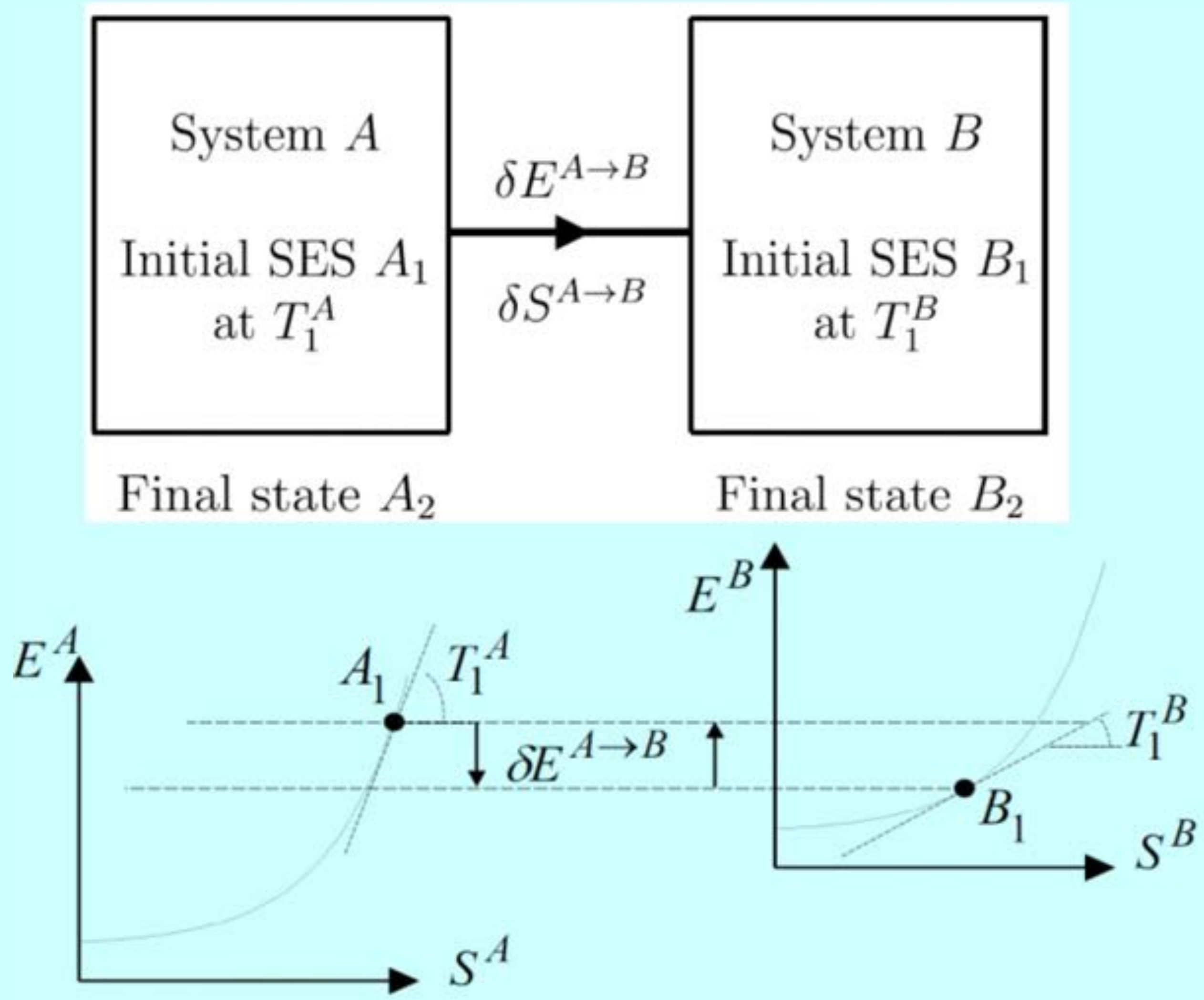
Of course, if you are unhappy with the recording you made or if it exceeds 5 min, you may try again until you are finally happy with what you submit.

Then, upload it in your Google Drive or other similar webservice and send me an email with the link for me to download your video by just one click.

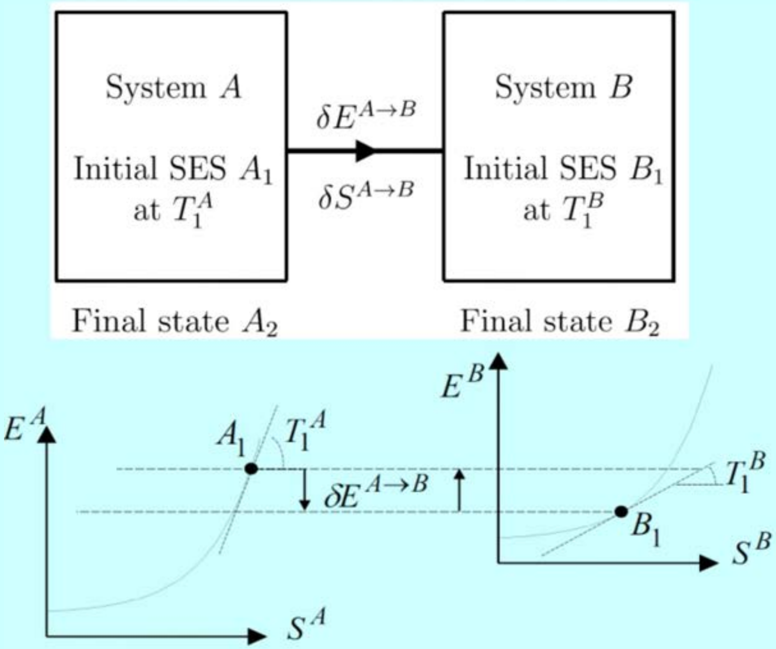
IMPORTANT: please make sure I can download it with no need for me to register for your favorite file exchange service (as I do not want to). Thanks.

Review of basic concepts: **nonwork interactions**

proof of Clausius statement of the Second Law (1/6)



proof of Clausius statement of the Second Law (2/6)

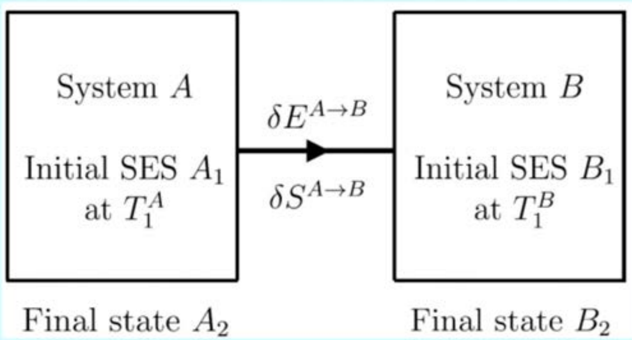


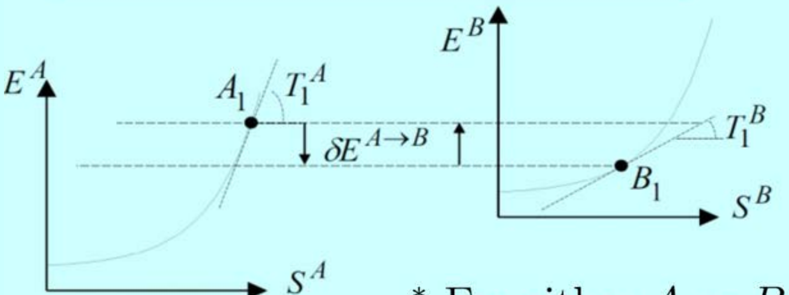
Energy and entropy balances for A and B:

$$dE^A = -\delta E^{A\to B} \qquad dS^A = -\delta S^{A\to B} + \delta S^A_{\rm irr} \qquad \delta S^A_{\rm irr} \geq_{1A} 0$$

$$dE^B = \delta E^{A\to B} \qquad dS^B = \delta S^{A\to B} + \delta S^B_{\rm irr} \qquad \delta S^B_{\rm irr} \geq_{1B} 0$$

proof of Clausius statement of the Second Law (3/6)





* For either A or B: $E_2 = E_1 + dE$, $S_2 = S_1 + dS$,

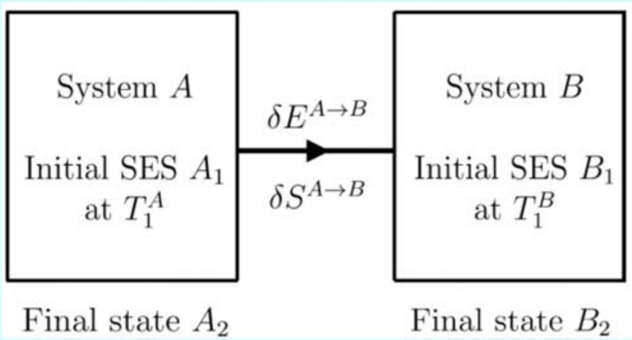
$$S_2 \le S_{2,\max} = S_{\rm SES}(E_1 + dE, V, n) = S_1 + \frac{dE}{T_1} + \left.\frac{\partial^2 S_{\rm SES}}{\partial E^2}\right|_{E_1} \frac{(dE)^2}{2} \cdots$$

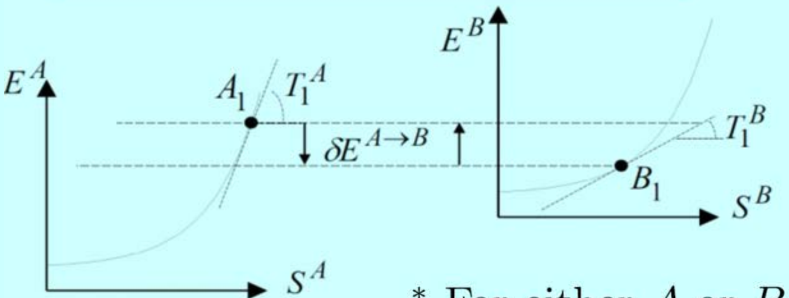
Energy and entropy balances for A and B:

$$dE^A = -\delta E^{A\to B} \qquad dS^A = -\delta S^{A\to B} + \delta S^A_{\rm irr} \qquad \delta S^A_{\rm irr} \ge_{1A} 0$$

$$dE^B = \delta E^{A\to B} \qquad dS^B = \delta S^{A\to B} + \delta S^B_{\rm irr} \qquad \delta S^B_{\rm irr} \ge_{1B} 0$$

proof of Clausius statement of the Second Law (4/6)





Principle of maximum entropy and fundamental relations of A and B:*

$$dS^A \underset{2A}{\leq} \frac{dE^A}{T_1^A} + \left.\frac{\partial^2 S_{\rm SES}^A}{\partial E^2}\right|_{E_1^A} \frac{(dE^A)^2}{2} \cdots \underset{3A}{\leq} \frac{dE^A}{T_1^A}$$

$$dS^B \underset{2B}{\leq} \frac{dE^B}{T_1^B} + \left.\frac{\partial^2 S_{\rm SES}^B}{\partial E^2}\right|_{E_1^B} \frac{(dE^B)^2}{2} \cdots \underset{3B}{\leq} \frac{dE^B}{T_1^B}$$

* For either A or B: $E_2 = E_1 + dE$, $S_2 = S_1 + dS$,

$$S_2 \leq S_{2,\max} = S_{\rm SES}(E_1 + dE, V, n) = S_1 + \frac{dE}{T_1} + \left.\frac{\partial^2 S_{\rm SES}}{\partial E^2}\right|_{E_1} \frac{(dE)^2}{2} \cdots$$

Energy and entropy balances for A and B:

$$dE^A = -\delta E^{A\to B} \qquad dS^A = -\delta S^{A\to B} + \delta S_{\rm irr}^A \qquad \delta S_{\rm irr}^A \underset{1A}{\geq} 0$$

$$dE^B = \delta E^{A\to B} \qquad dS^B = \delta S^{A\to B} + \delta S_{\rm irr}^B \qquad \delta S_{\rm irr}^B \underset{1B}{\geq} 0$$

proof of Clausius statement of the Second Law (5/6)

Energy and entropy balances for A and B:

$$dE^A = -\delta E^{A\to B} \qquad dS^A = -\delta S^{A\to B} + \delta S^A_{\rm irr} \qquad \delta S^A_{\rm irr} \ge_{1A} 0$$

$$dE^B = \delta E^{A\to B} \qquad dS^B = \delta S^{A\to B} + \delta S^B_{\rm irr} \qquad \delta S^B_{\rm irr} \ge_{1B} 0$$

Principle of maximum entropy and fundamental relations of A and B:*

$$dS^A \underset{2A}{\le} \frac{dE^A}{T_1^A} + \left.\frac{\partial^2 S^A_{\rm SES}}{\partial E^2}\right|_{E_1^A} \frac{(dE^A)^2}{2} \cdots \underset{3A}{\le} \frac{dE^A}{T_1^A}$$

$$dS^B \underset{2B}{\le} \frac{dE^B}{T_1^B} + \left.\frac{\partial^2 S^B_{\rm SES}}{\partial E^2}\right|_{E_1^B} \frac{(dE^B)^2}{2} \cdots \underset{3B}{\le} \frac{dE^B}{T_1^B}$$

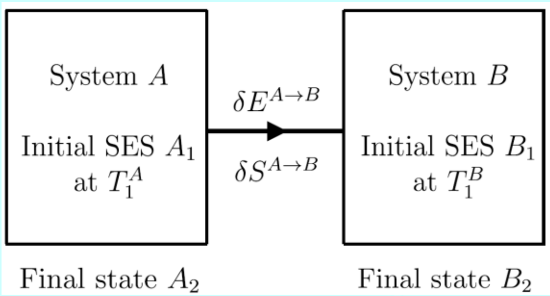
Combine the above (eliminate dE^A, dE^B, dS^A, dS^B):

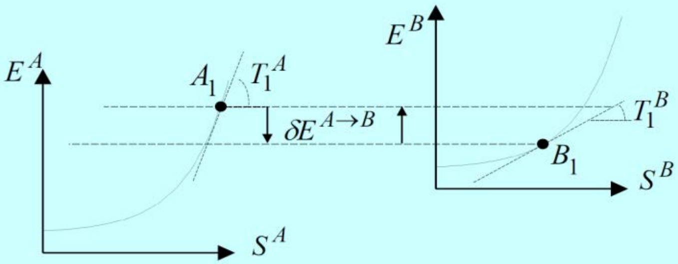
$$-\delta S^{A\to B} + \delta S^A_{\rm irr} \underset{2A,3A}{\le} -\frac{\delta E^{A\to B}}{T_1^A} \qquad \delta S^{A\to B} + \delta S^B_{\rm irr} \underset{2B,3B}{\le} \frac{\delta E^{A\to B}}{T_1^B}$$

Solve for $\delta S^{A\to B}$:

$$\boxed{\frac{\delta E^{A\to B}}{T_1^A} \underset{2A,3A}{\le} \delta S^{A\to B} - \delta S^A_{\rm irr} \underset{1A}{\le} \delta S^{A\to B} \underset{1B}{\le} \delta S^{A\to B} + \delta S^B_{\rm irr} \underset{2B,3B}{\le} \frac{\delta E^{A\to B}}{T_1^B}}$$

proof of Clausius statement of the Second Law (6/6)





Energy and entropy balances for A and B:

$$dE^A = -\delta E^{A\to B} \qquad dS^A = -\delta S^{A\to B} + \delta S^A_{\rm irr} \qquad \delta S^A_{\rm irr} \underset{1A}{\geq} 0$$

$$dE^B = \delta E^{A\to B} \qquad dS^B = \delta S^{A\to B} + \delta S^B_{\rm irr} \qquad \delta S^B_{\rm irr} \underset{1B}{\geq} 0$$

Principle of maximum entropy and fundamental relations of A and B:*

$$dS^A \underset{2A}{\leq} \frac{dE^A}{T_1^A} + \left.\frac{\partial^2 S^A_{\rm SES}}{\partial E^2}\right|_{E_1^A} \frac{(dE^A)^2}{2} \cdots \underset{3A}{\leq} \frac{dE^A}{T_1^A}$$

$$dS^B \underset{2B}{\leq} \frac{dE^B}{T_1^B} + \left.\frac{\partial^2 S^B_{\rm SES}}{\partial E^2}\right|_{E_1^B} \frac{(dE^B)^2}{2} \cdots \underset{3B}{\leq} \frac{dE^B}{T_1^B}$$

Combine the above (eliminate dE^A, dE^B, dS^A, dS^B):

$$-\delta S^{A\to B} + \delta S^A_{\rm irr} \underset{2A,3A}{\leq} -\frac{\delta E^{A\to B}}{T_1^A} \qquad \delta S^{A\to B} + \delta S^B_{\rm irr} \underset{2B,3B}{\leq} \frac{\delta E^{A\to B}}{T_1^B}$$

Solve for $\delta S^{A\to B}$:

$$\frac{\delta E^{A\to B}}{T_1^A} \underset{2A,3A}{\leq} \delta S^{A\to B} - \delta S^A_{\rm irr} \underset{1A}{\leq} \delta S^{A\to B} \underset{1B}{\leq} \delta S^{A\to B} + \delta S^B_{\rm irr} \underset{2B,3B}{\leq} \frac{\delta E^{A\to B}}{T_1^B}$$

Consequences of

$$\frac{\delta E^{A\to B}}{T_1^A} \leq \delta S^{A\to B} \leq \frac{\delta E^{A\to B}}{T_1^B}$$

Clausius statement:
$\delta E^{A\to B} \geq 0$ only if $T_1^A \geq T_1^B$

Heat interaction:
in the limit $T_1^A \to T_Q \leftarrow T_1^B$

$$\delta S^{A\to B} = \frac{\delta E^{A\to B}}{T_Q}$$

* For either A or B: $E_2 = E_1 + dE$, $S_2 = S_1 + dS$,

$$S_2 \leq S_{2,\max} = S_{\rm SES}(E_1 + dE, V, n) = S_1 + \frac{dE}{T_1} + \left.\frac{\partial^2 S_{\rm SES}}{\partial E^2}\right|_{E_1} \frac{(dE)^2}{2} \cdots$$

