

GANG CHEN: I have a set of handouts. If you don't get a copy, it's all on the web. And if I do this well, every time you should get a handout. And my objective is for you to focus on listening and discussion, not to take notes. So it's a [? similar ?] handout, or you go-- it's online.

It's an experimental course, the first time I'm offering it. So it has a special, say, 997 number, 6 units. If you look at the handout, if you print it out yesterday, I changed something and one of the changes is very important. This course will be pass fail basis. And what does it mean is if you are undergraduate student, for example, in course 2, pass fail could count for unrestricted elective. But it does not count for restricted elective.

And if you are course 2a, it does not count for your concentration. And if you are a graduate student, this is another entry level course. It's not an entry level course. So those are just for your clarification. And if you look at the handout, the syllabus, what we will have is once a week at this time. And also, I have office hour. And I have volunteer TA. Daniel is here. You can contact him if you have questions. If necessary, we'll ask Daniel also set up an office hour.

And what else? I don't have a copy in my hand. Thank you. In terms of the requirement homework, the homework is every week I will assign one reading. And you will turn in your report, one page summary of the reading. You can summarize what you understand, what you do not understand. And that's the-- I do not see will give an extra number of homeworks. So it's a reading based homework.

And at the end of the semester, you will write a final report, project report. And the detail of the report will answer in the middle of the semester. Any questions? OK if not, I will start. Like I said already, everything is online. And you can jot down whatever notes you want. But my purpose is not to have you write too much and focus on this. I do not have a textbook on this.

So let me start. So this course, if you look at the title, is a direct solar thermal to electric energy conversion technologies. There are few points I want to emphasize. One is a heat-based course. Solar thermal, this is all focused on the thermal aspect. The solar energy is, in this course, is a big, giant heat source.

And the other aspect important in this course is the direct energy conversion. So the focus is on how to convert the heat into electricity directly without the moving parts. And when I say the technology, you realize that we move on. Those technologies, some already existing. But clearly is not widely spread used. It's not widely used.

So they are still under development. And whether they will work out or not down the road is not completely sure. But that's your opportunity. You're here to do research and future development. And maybe you will push some of this technology to real world.

So I will start with this slide to make a case that it's important that we think about the heat. You can start either from the input side, look at how we are using all the energy. Fossil fuel or even nuclear, the energy source from the energy source to convert this energy into electricity, into mechanical energy. You can see more than 90% is [? VIP. ?] Mostly is thermal mechanical means to convert the energy into mechanical energy or electrical energy.

And the other aspect is look at the end use, how much energy is lost. And you can only see the numbers here. You'll find out that more than 60, about 60% to 70% of the energy are wasted. So what are we are looking into is whether we can develop alternative ways to either use the input energy or convert some of the waste energy into electricity. Yes.

STUDENT: Are there 10% missing or [INAUDIBLE]

GANG CHEN: 10% missing, you have to look at some of this in here. We're talking about heat sources. Just a few examples. If you look at the automobile, the driving efficiency as a measurement is less than 20%. And if you want to remember, it's easy to say about one third of the energy goes through the tailpipe exhaust. The other third roughly goes to the radiator.

Those are completed with-- so can we recover some of this energy? And in fact, this is challenging because it's a model. You have to feed into the compact systems. And in those cases, it turns out people look at various options by say the mechanical way of converting those energy into electricity is not very feasible. So there are a lot of companies looking into thermoelectric, for example.

And now you look at another way that we use energy is buildings. And give me an example. In New England in the winter, home furnace, boilers, most of the time they are off. Very often, they are on. And there's a combustion process. And [INAUDIBLE] what we're using for-- hot shower, heat up the room. That's a low quality source.

Once you switch to this temperature, you have high temperature combustion. You quench to low temperature just take a shower. And you are generating a lot of entropy. And in some regions, there are big coal fired power plant or oil fired power plant. In this case, the people take the steam and supply the heat to the community. That's electricity, heat cogeneration, cogen plant. But we don't do this at home.

The technology is not there yet. People are still-- people are pushing. And in fact, there is a, say, [? Honda, ?] is fighting, for example, developing home stand-alone internal combustion based system. But it's still not taking advantage. During the combustion, during the heat of the hot water process, we are really generating a lot of entropy. We are wasting useful energy.

Can we develop technologies that can do home cogeneration? Industrial process, that's about another third of the energy that's used. And their heat sources are different temperature, where wide temperature range. Once I visited the aluminum smelting plant. So that's just converting aluminum oxide into aluminum. And the process is a high temperature electrochemical process.

So you heat up aluminum oxide to molten state. And then you use a electrochemical way to extract the aluminum. And those plants are typically built just beside power plants, the electrical power plants. They travel the highways. There's a 500 megawatt electrical input. That's a huge power generation, power requirement.

Just give you an example, how much is a 500 megawatt? A typical power plant is about a one gigawatt. And the world solar installation, solar photovoltaic installation, it's about six gigawatts. So one power plant, one aluminum smelting plant, takes 500 megawatt.

And, of course, there is a part of this aluminum oxide to aluminum energy stored as chemical energy. But the half of this is dumped into our environment as heat. And right now there's no technology to really recoup this into anything useful. So now, you think about this is a waste heat, renewables. When we think about the photovoltaic solar, we often think about photovoltaic. But fundamentally, solar is a high temperature heat source.

Anyone knows the equivalent temperature of the solar radiation coming to Earth?

STUDENT: 20,000.

GANG CHEN: Close.

STUDENT: 7, 5, 7,000 kelvin.

GANG CHEN: 6,000 kelvin, 5,800. And geothermal. Those are renewable. Those are heat sources. So how we can use as a heat? And in fact, if you look at how people, what people are doing now, the solar utilization, the biggest installation is in solar water, not in [INAUDIBLE], just hot water.

And I came from China. When I grew up, there was no hot water. [INAUDIBLE] didn't sell them. I take a shower. No water in the winter.

And nowadays when I go back home, we have solar hot water. And it was a really fundamental life changing technology, very cheap. But it's actually high-tech.

So here I'm showing the solar hot water system that's widely used in China. And give us a order of magnitude, it's about 100,000,000 square meter installation. 100,000,000 square meter.

And if you take 100,000,000 square meter, you take a solar energy. One square meter is about one kilowatt. And thermal efficiency, they collect the heat around 60%. So you multiply this together. How much you get? You get about 60 gigawatt solar energy being used.

And I said as a high-tech, this is a [INAUDIBLE] tube-- vacuum tube. And so there is an inner layer, outer layer. And in between is a vacuum tube. And in the inner layer tube, there's a coating. This coating will absorb the solar radiation, but the minimize the thermal radiation. And it's done very cheaply. In a typical tube like this high, it's only-- I waste of the ones that are factory, \$1.5 a tube, [INAUDIBLE], all manufactured.

So because of the really the low cost, it's widely used in developing countries. And of course, you can talk about the electrical generation. This is their installation in US Kramer Junction, where the heat from the sun is a focus through those parabolic troughs. Heat up the fluids. And these fluids are used to drive steam turbines.

And there are also the power system that people are developing. Some are used in Spain, and I think also Algeria, where the mirrors focus and heat up the power here. And this goes to a higher temperature. And the conversion is we have mechanical systems.

And the question, [INAUDIBLE] here in this course will not-- as I said, we're not looking into mechanical energy conversion. We're looking into alternatives. And I just want-- so some of those alternatives that we're going to discuss, one is a thermoelectric. In fact, Daniel, does that work? If you can turn it on. So you will see that this is a conversion that, as long as there's a temperature difference, you can generate electricity. It's by solid.

If you have used thermocouples, those are essentially thermocouples putting together. Thermocouple, of course, we just want to generate an electrical signal. But in this case, you can also, if you make the efficient, you convert heat into electricity.

The second-- here, the working fluids is electron. And in this demo, what we have is just a copper. Here is a heat source. Copper coming out heat to this side. The top side is hot. And there is this thermoelectric device, very thin. It's about two millimeter, three millimeter sandwiched between this copper and the heatsink, aluminum block.

And hopefully I don't see it. But once the temperature difference, there a difference, you'll start to see the LEDs lighting up. So that's direct, no moving parts, using electronics as working fluids. And the second example is thermionic energy conversion.

How many of you know that in our department there is a microfluidic catabolism plant? Fabulous. That's on the second building 3, second floor. [INAUDIBLE] was a professor in the mechanical engineering department, 1950s. And he developed this thermionic engine. So you can see it lighting up.

And of course, you have to maintain this temperature difference. So if we burn it for a long time, the other side, the cooling is not there. The temperature difference will be becomes small. Your power, I saw it before, it actually fluctuated before. But this is also another important aspect.

It's a thermal system. So you have to put the heat in, take the heat out. And the converter is small, but your thermal [? auxiliary ?] system may be very big. So you have to do a good engineering design. I was mentioning thermionic energy emission and energy conversion. And Professor [INAUDIBLE] started his PhD on thermionic engines. And it's been off thermo electron.

If you drive 128, you probably see that [? component. ?] Thermo electron is called thermo feature. And the original purpose was to develop the thermionic engines. In fact, if you go to read it, it's 1956 or 58, I forgot. In the 1950s and 1960s, the efficiency was about 18% from heat [INAUDIBLE]. But it's a very high temperature. So there are a lot of technological difficulties and at the end they didn't pursue in terms of commercialization.

Of course, photovoltaic, normally we don't think of it as a heat engine. But it is a heat engine. It's limited by the second law of thermodynamics. And in fact, any time we can get close to the second law, you'd be really happy. But see, this is, of course, using the photon from the sun.

And there's another way is using the photon from a [INAUDIBLE] or a resource. You have a home furnace. And you can use the photon from there to put a photovoltaic. That's called a thermophotovoltaic.

So those are all ways some of the direct energy conversion, no moving parts here is using the charge as a fluid. And we'll be talking some of those technologies and the basic principles. And of course, when we think about the solar, the advantage compare potential advantage-- doesn't say it's demonstrated fully yet-- is that in the case of photovoltaic, only the energy above the bandgap of semiconductor useful.

But when you think about solar thermal, you can think about the full spectrum of the solar energy. So this year we started actually department energy center looking into how we can advance this technology. What is solar thermal photovoltaic? So here, the idea is I take the solar energy, absorb it. This is not directly onto photovoltaic cell. But they absorb it to heat up this and re-emit the photon.

Why do you do that? Because when you re-emit the photon, if you can do better control, you can emit photons right at the bandgap of the photovoltaic cell. And theory says you can potentially get a really high efficiency with a single junction cell, but no demonstration. So that's one of the direction we're looking in this center. And the other direction is looking into thermoelectric, just like what we have here. The temperature difference, instead of the burner here, using this anytime you have temperature difference.

And again, the k is how we can make this efficient, how we can make that low cost. So when you think about energy, the cost is the main concern. The photovoltaic cost right now is just too high. So most people are reluctant to put on their rooftop. So that's the motivations for this course.

And to really understand those technology, we'll have to dive into, for some of us-- [INAUDIBLE] For some of us, this may be unfamiliar, territory, particularly, I think for mechanical engineers, because most of the time, we are familiar with heat engines using steam, using molecules. And here we're talking about heat engine using electrons and solid.

So we'll have to cover some of the basic background knowledge in this area and understand the technology, understand the limitations, and I want to bring you to the forefront of research. What are people doing? Maybe you have new ideas in doing new things.

Now, just make another comment, this is very different. You can say the dying down process is slow. Thermal has a potential to do storage. With a thermal mass, you can store the energy. A photovoltaic will take if there's no light emitted, don't have any power in it. So that's one potential advantage of thinking thermal.

OK, so now I'm going to move into really-- the first what I want to use here is to review some thermodynamics and the heat transfer. And if you have taken thermodynamics, you will know some of the material. If you are from other department like physics, you also know some material. Most groups have some material, maybe enough material. But my advice is just to take it. This is, I would say this is my starting point for future discussions.

So when we think about the first law, with thermodynamics we think about two laws-- First law of thermodynamics, second law of thermodynamics. We take a system and we define by defining the boundaries. So the boundary define our system. And across the boundary we have heat flow. We have work flow.

And typical convention is that they say heat goes in. We take the positive. And work goes out. We take as a positive. That's just due to convention. If you get a negative sign, it means your heat goes out or work goes in. And then we have closed system means that there is no mass flow across the system boundary, or open system when there is a mass flow across system.

In the simplest form of thermodynamics, the first law is just the energy balance. On the left hand side, I have the energy of the system, the energy change. And on the right hand side is the heat transfer into the system minus the work output. Simple energy balance. You can write it in the differential form or you can write into [INAUDIBLE] form. And then you do heat transfer. You actually use the [INAUDIBLE] form most often.

See, some of you may still remember what's important is to recognize on the left hand side is a state property that does not depend on the process. On the right hand side is a process dependent. So it's actually very amazing. You take two process dependent quantity. You take a difference. You get a quantity that's independent of the process. Looks very simple. But this is a truly important aspect.

And the energy of the system, including kinetic energy, potential energy, and internal energy. Or if you have other forms of energy, like elastic, magnetic, or those. And consider your relation that we're using is specific heat, for example.

And here the specific heat is the temperature derivative of the internal energy. Here, the internal energy, I use a small [INAUDIBLE]. It could be either per unit mass based or per unit of volume based.

So, that's the first law. Second law. Second law we're written into our equation. But it's not an equation. It's an inequality. And what we have here is the entropy change. Again, this is a state property, entropy, equals the entropy transferred across the system boundary and pass entropy generation in the system.

On the right hand side it's process dependent. And on the left hand side is state properties that's independent processes. Now, I think about the heat engines. The heat engine, which when we think about engine, we think about cyclic motion. You don't want to design engine. You only go one direction, doesn't come back.

And so when we think about cyclic motion during a cycle, any state property, go back to the original, the independent process. Once it goes back, the change is 0. So here, we just wrote the entropy and energy, and other state properties. So if the entropy change is 0, now what's the maximum efficiency you can get?

And maximum happens when there's no entropy generation. So if there is no entropy generation. [ΔS_{SG}] is 0. And I look at this is my system. The heat is positive. Heat output is negative. So I have my second law written into left hand side is 0. Entropy generation is 0. This entropy transfer across system boundary.

And this gives me a relation between the heat in and heat out and temperature of the two reservoirs. And if I use this relation, I go to calculate what's in this case, what's the efficiency of this heat engine. You can see the efficiency, of course, we define as the work we get versus the price you pay as the heat in. The efficiency here is the work.

Now, the work first all tells you the work equals heat in minus heat out because the internal energy during a cycle change is 0. So you can see you've got a [η_{power}] efficiency. That's the maximum you can get for any heat engine operating between two constant temperature [INAUDIBLE].

People sometimes look down into heat engine. It's a heat engine, meaning it's thermal dynamic. It's not-- you can't convert all the heat into electricity. That's correct, unless you have 0 Kelvin. But the important thing is that you put in the lumber and you see where we are. The heat engine is very [$\eta_{\text{respectful}}$] if you have even just a 200 degrees temperature heat source and if you can get [INAUDIBLE] efficiency-- 40%.

If you take the sun, 95%. And then more than thermal power plant is about 40%. I said automobile internal combustion engines. The driving efficiency is less than 20% and the AC engine efficiency is probably around 25%. So heat engine, if you can achieve the theoretical efficiency, you're doing really well.

The problem is there are a lot of losses that limits us to get to practical [INAUDIBLE], practically get too close to the economy efficiency. So that's what you learn in 2005 thermodynamics.

Now I want to go to a little bit microscopic picture. Because what's an entropy, for example? And so let's look at an isolated system. An isolated means there is no heat transfer across system boundary, no work transfer, no mass transfer. It's isolated.

And I'm going to look at, imagine I have the magic eye, can look at the individual molecules, the electrons in the system. And there are many different possible configurations. A molecule, so the molecules in the system could be at different states, different position. And each atom could have many different possible energy states. This is from quantum mechanics.

So every possible configuration possibility-- you can go back and take a two color, two different color beads and mix them up. That's your experiment on this micro picture. And every possible configuration is a micro state. And let's suppose we know how to count what's the total number of those possible configurations. So the total number, that's omega microscopic state. Those are very big numbers.

Now, what are the basic principle of this microscopic picture says every state that is equally possible, equal rights, no discrimination. And so this is the equal probability principle. Every micro state is equally possible. And it's based on this. All the statistical description of the statistical thermodynamics is based on, starting from here.

And what Boltzmann did is to relate this number of total microstates to interpol. This is actually in Boltzmann tombstone. It goes to Vienna. And this k_B is the Boltzmann constant, 1.38. I don't remember any-- people in physics, they use atomic units. I always remember things in the SI unit. So that's a difficult large exponent, 10^{-23} to the minus third [? jar. ?]

That's good to keep some numbers in mind. So that's the, like I said, a starting point for the microscopic description. And here, we'll consider an isolated system. And now if you think about a not isolated system, a system that can thermally [INAUDIBLE] exchange energy with the environment. And because of that, each micro state is no longer have equal right in terms of observation.

The probability of this system, if you can observe this system in certain specific micro state that has certain energy, e . And in this case, this is no longer equal probability. And we're not going to derivation. But you can go from here, equal probability, to construct an isolated system that made up your system and the environment. And with all that derivation at the end. We get the probability of the system actually inserting one micro state. This is the one micro state that has an energy, e . It's depending on the temperature of the system and the energy here, [? exposure. ?]

So you can see higher energy, less probable. And if you do chemical combustion or a lot of Arrhenius law you probably learned in high school, that's the exponent. That's coming from here, [? personal ?] statistics.

And so here is a closed system. What if there is a mass flow, particle flow across system boundary? And in that case, this is similar. But there is an extra chemical potential uphill in the exponent.

And this is a constant temperature, but open system. And μ here is a [? chemical ?] potential. What is [? chemical ?] potential? Think about temperature. Temperature is a driving force for heat flow. [? Chemical ?] potential is the driving force for mass diffusion, mass flow.

Pressure is the driving force for mechanical motion. Those are all thermodynamic, intrinsic thermodynamic quantities-- pressure, chemical potential, temperature. The other way to think about the chemical potential is it's the average energy needed to add a particle into a system or take a particle out of the system.

So first chemical potential is the energy look at. This is the equivalent here. And second is driving force for the mass transfer. So if you do, say chemistry, you do mass diffusion or you do batteries, this is a quantity you will be dealing with all the time.

So this is a microscopic. And now I'm going to apply what I just wrote down in the previous slide to molecules. I have lots of molecules. And I have an equilibrium, temperature t . I think the one molecule. It has a kinetic energy, a single, simple kinetic energy of the molecule. So it's $\frac{1}{2} m v^2$. [INAUDIBLE], molecule is the velocity is the three components, the v_x^2 , v_y^2 , v_z^2 . So that's what I have here.

And this is the energy. Remember, in the previous slide we say this is now a constant temperature molecule in thermal equilibrium at constant temperature. So that's the statistical system I'm using. And this is the energy that's a kinetic energy. And here is a probability, probability I have a normalization factor.

So I put in the probability of this molecule having the velocity v_x , v_y , v_z . And the next step I want to determine a , determine that pre vector, a . What I do?

The probability-- so the molecule probability is the velocity is from minus infinity to positive infinity. I include every possible chance. So if I equal the error possible chance, that's my normal, the way I find out this factor a . That's all, say, velocity from negative infinity to positive infinite. And this should sum up to 1. They've got to have the velocity in between these. And if you do this integration, you find your factor, a .

And this is the famous Maxwell distribution for molecules.

STUDENT: It should be a minus.

GANG CHEN: Thank you. Yes.

STUDENT: [INAUDIBLE]

GANG CHEN: There is a minus. There's this minus sign here. Thank you. And see the other one. Yes. Please add [? another ?] [? 7. ?] And here it's the same problem. I just copied.

So now if it's a probability, once you have a probability, you can calculate your observation. What's the average once you have probability? So I want to find the average energy, kinetic translational energy motion, kinetic energy of this molecule. So here is the energy weighed by this probability over all possible velocities.

And again, it's an integration. You can do yourself if you look at a table. And you find out is a very simple result e equals two thirds, $\frac{3}{2} k_B t$ [INAUDIBLE]. It's $\frac{3}{2} k_B t$, actually. It's a very good language to remember, $k_B t$. But also it's a fundamental. There is a fundamental principle, equal partition principle, which says if your microscopic energy is quadratic-- so if you look at this, kinetic energy is quadratic in velocity and v^2 .

So if energy is quadratic, each of these quadratic terms contribute to average energy half $k_B t$. So if you look at this molecule there, because it has three quadratic terms, so each contribute one part $k_B t$ in total [INAUDIBLE]. So this is the equal partition principle.

And now I would like you to get an idea of what is $k_B t$. This is really something that if you want to have microscopic mind, you should remember. How large is it? $k_B t$, 1.3 times [INAUDIBLE] times third, 300 degrees Kelvin. I think I forgot to correct this again. This should be 4, not 5-- 4.14.

4.14. And if you take a 4.14 divided by 4, that's a big number. Too hard to remember. And people-- again, different field people use different ways, different measures. And one often used the unit is electron volt as an energy unit. What is the electron volt?

You have a charge. You apply an electric field, one volt, say under one volt. That's a potential how much energy it has. So one electron volt is 1.6×10^{-19} [? JR. ?] So I translate this into a room temperature one kbt 26 literally electron volts.

Now, think about if this is the average energy of the molecule, what's how fast they are moving? So the molecule moving in this room. You can go to estimate, because this energy equals $\frac{1}{2} mv^2$. So you can go to estimate the velocity of the molecule, one half cancel.

You do the math. I took the oxygen as an example. And all the number of oxygen atomic number is 8. So it's 16 neutron proton. And each of the proton mass is 1.677 plus 27.

So number is about 200 meter per second. Depends on whether it's a light molecule, heavy molecules. So those are the microscopic molecular picture. That's about molecules.

Let's think about the electrons. Molecules, what we see, we did not do anything quantum side. The Maxwell distribution is a classical statistical thermodynamics. Now, when I do electrons, the quantum mechanics sometimes is very important. So it's the more common picture.

From quantum mechanics, we learn-- you have a physics before, basic idea. I'm sure you all heard about energy levels of quantum mechanics. And some of you may still remember, if you have quantum mechanical state, you can only have one electron-- Pauli exclusion principle.

And the quantum mechanics, actually here I have listed the quantum Einstein relation that gives the energy. This actually started from photons. So next slide I have more energy related to frequency of the photon and the momentum relates to wavelengths of the photon. So that's the so-called the Planck Einstein relation.

And again, this is another number. If you think about the quantum, you always see h Planck constant at 6.6×10^{-34} [? jar ?] [INAUDIBLE].

Now I'm going to look at one quantum mechanical state. And this quantum mechanical state can have maximum of one electron. It can have 0 electron. You have one electron.

And this quantum state has an energy, e , quantum mechanically allowable energy. When the electron is there, this state has energy, e . When it's empty, the electron, the actual energy of the system is 0. But I only have two possibilities. So I have-- there's two possibilities. I sum this up. I want to find this a , the difference between the molecule I did before.

And here, for molecule velocity I say, OK, from minus infinity to plus infinity, I include everything. But for the electron here, I only have 0 electron or 1 electron. So I sum this up to what I will find a . I'm not going to detail the math. So from the previous relation, I found a . And now I give you an average. Again, I can give you the average energy. Because if I do observation, it's 0 or e . And if I do a lot of observation, it's a equilibrium temperature t , what's the average time, average energy, or average number of electrons?

Each individual observation could be 0 or 1. But there is an average possibility. So if I look at the average, again, say this is an n could be 0, 1. And they sum of two Fermi-Dirac distribution. You look at that. What's the range of this $\langle F \rangle$ the average?

STUDENT: [INAUDIBLE]

0 to 1. That's so they can't be above 1. Its maximum is the 1. Minimum 0. So it's between 0 and 1. And then you plot this depend on the temperature. And it depends on whether the energy of this level is above the chemical potential or below the chemical potential.

The chemical potential is they say, we said before, now you can think of this as a one quantum state that the open state system, the electron can come in and goes out. The chemical potential is what's the average energy you need to take the electron in or the electron out-- take the electron out for put electron in.

So depends on whether it's above or below. You can say it's close to 1 or close to 0. Very narrowly distributed near the chemical potential. And in fact, if your [INAUDIBLE] is 0, this will be a sharp position. At the chemical potential above is 1, here below. Let's say above 0 and below 1. And the chemical potential at 0 temperature, there is a special name people give often called that Fermi level.

But in a lot of different fields, sometimes people just call Fermi level for energy transfer. It's terminology convention. So that's for electron.

Now we're going to lecture two. We talk about molecule. We'll talk about electron. Next I'm going to talk about electromagnetic wave, the photons.

I mentioned before, the Planck Einstein relationship is really looking into photon. And later on was generalized for any material wave. And so what we have is if there is electromagnetic wave at a certain frequency, μ , what Planck found is that the energy of that μ , the quantum mechanical state-- at the time there was no quantum mechanics. So electromagnetic mode, of specific mode, the energy has to be quantized. And basically energy quanta is $\langle h\mu \rangle$.

And this other quantum [INAUDIBLE] was called a $\langle$ photon. $\rangle$ And when Einstein came in, he's saying that the quanta not only just have the wave, but also has a momentum. So this is where the wave particle duality came from. So the momentum of the quanta is related to the wavelength of the wave, and again, the edge.

And so each quantum state, the energy can only be multiple of the edge μ . Normally when we think about the continuum mechanics can be any value. But the important difference is that the energy could be only multiple. So n is now integer. And it has a one half. That's also called this one half. Normal is not important. It's called 0 point energy.

And it's really-- if you still recall from quantum mechanics, that's the so-called the Heisenberg Uncertainty Principle requirement. So this is 0 point energy. Normally it's not. So I'm not going to give much.

So that's a photon. Now I'm going to-- it turns out the photon, you can think of this as the electromagnetic wave. Now I'm going to talk about different waves. That's the lattice wave, the atomic vibration. The [INAUDIBLE] vibration now, you can think about starting with classical mechanics. If you have a mass spring system, the fundamental natural frequency is square root of k over m .

And, of course, in classical mechanics, we see the energy is related to the amplitude of the wave, the velocity. And once you go to quantum, it turns out this way, the energy cannot be any value, same as the electromagnetic wave. And the energy can only be multiples-- same, you can say multiples of $h\nu$.

And now there was an analogy made for this compound of vibration. That analogy is [INAUDIBLE]. Electromagnetic wave, the basic energy quanta is the photon. That is atomic vibration or the energy based energy quanta is photon.

OK, so what's the statistic they obeyed? At a certain quantum state, if you think about electrons, could be 0 and 1. Maximum is 1. And here, at the mole, at this frequency, it could be 0, 1, to infinite. So that's a difference. Electron, 0, 1. And here, it could be any number of integer numbers.

So I need to find the distribution average number of photon or [INAUDIBLE] if I have a equilibrium system. Do the same thing. I have the probability.

And now, if I have n photon or [INAUDIBLE] in this microstate, the quantum mechanical state that has this specific frequency, that's the probability. I sum this up from n equals 0 to infinite. That still should give me 1. You have either 0 or 1, or infinite number, or the in between, somewhere in between.

So sum up, I have one. I will determine a . And this a , from this a , I go to find the average again. Once I find the probability, I find the average number same way as I did for electron. And this average number now is the Bose-Einstein distribution. And this Bose-Einstein distribution, if you compare it with the previous Fermi-Dirac distribution, the only thing changed is a plus sign now goes to minus sign.

And because of that, it's no longer between 0 to 1. It's between 0 to infinite. And in fact, for photon and photon, because the number is not conserved. It's not like the electron is conserved [INAUDIBLE]. So chemical potential concept is not [INAUDIBLE]-- is not a good one. So that's a 0. We take it to 0. And this is the distribution for the Bose-Einstein distribution.

STUDENT: What's the reason for setting μ at 0?

GANG CHEN: It's not a conserved quantity. It's, say, when we talk of mass transfer particle, that's a fixed number. Total number either going out or going in. But that's a conserved number. Here is another number.

But there are actually-- see, later on when we talk about the solar cell, you'll find out there is a quasi Fermi level getting into here. So it's actually a very deep similarity. So this is just a simple argument.

That's about-- now, in this time frame, like I said, you're probably familiar with the classical thermodynamic. You're now familiar with statistical side. Like you just take it. This is all the result we'll be using when they get into [INAUDIBLE]. Then microscopically we have [INAUDIBLE] distribution. We have Fermi-Dirac distribution for electron. We have Bose-Einstein distribution for phonon photon. That's what you need to know.

And what we need, I'm going to summarize next. This is the heat transfer. Before it was thermodynamics. Now, heat transfer, again, you take the 2005, 2006. This is a very-- should be very familiar. We have three modes of heat transfer-- conduction, convection, and radiation.

And describing the conduction is a Fourier law of diffusion. So the heat transfer rate, here for the rate in watts, it's proportional to the area, proportional to temperature gradient. A proportionality constant is the thermal conductivity. And that's a material property. Normally, we think of it as a material property.

And the negative sign actually take care of the second law, says temperature, heat will flow from high temperature to low temperature. And you can do that. from watt you can do flux. That's a per unit area based. So just normalize the area. Or you generalize it to 3 dimension rather than writing into one dimension.

So that's [INAUDIBLE]. And convection is we have fluid flow in the surrounding of a heated or cold surface. The heat transfer between the surface and the fluid is proportional to the temperature difference of the wall and the fluid. This fluids are actually transition. The temperature goes from the wall to gradually to the center of the fluid or ambient.

And the proportionality constant here is h , the heat transfer coefficient. But h is a flow dependent. It's not a material property. It depends on how fast the fluid flow and depends on the geometry. So this is a big topic in 2005, 2006. And we know there is a natural convection, forced convection, just in terms of the terminology there.

So those are conduction and convection. And, of course, the thermal of heat transfer is radiation, thermal radiation. Thermal radiation for blackbody one single surface or black object. The emission is given by the Stefan-Boltzmann law as the proportional to temperature force power. The proportionality constant is Stefan-Boltzmann constant. So this is a for black object.

A real object is not black. So we use a emissivity to characterize it. And we'll discuss some more down the road. Now, if I have two objects, exchange heat, in this case, it could be vacuum. That's the difference of radiation compared to conduction and convection, which requires a media. And here, the heat transfer is by electromagnetic waves.

So I wrote a simple form of relating the heat transfer is the [INAUDIBLE] to the fourth power. And remember here, you have to use a Kelvin, not degree Celsius. Look at the units and so on in Kelvin. And if the object is not black, we use the emissivity. And here is the factor between two surfaces, because the two surfaces may be different. Like I said, we'll discuss more down the road.

So those are three modes of heat [? conduction. ?] And in this course, we'll talk more on conduction and we'll talk more on radiation. You have learned a lot. Some of you learned a lot of convection. So that's probably fear.

Now, let's think a little bit more on conduction. If I have a one dimensional multi generation inside, I can say heat flow is constant and secure is constant. You go back to the Fourier law. You integrate it. You find out that the heat transfer between the two points from this to the other side at steady state is proportional to thermal conductivity area inversely proportional to the length.

So if you want to do thermal isolation, what do you do? You use a long object. You use a small cross section. That's the thermal isolation. A lot of times when you design a thermal system, not only heat getting in, you also want to isolate it.

And we can combine all this right into thermal resistance. That's just a definition. Because this way, it looks very similar to electrical circuit where your driving voltage is now the temperature difference and your current is the heat current and your resistance.

So you can use the same way as you do circuit analysis if you know the thermal resistance. This is for different geometries, different. And even for convection, you can express, you go to do thermal resistance. So you can build some resistor network to do some simple heat transfer.

And another thing that's important to keep in mind is the order of magnitude. What are the good conductors? What are good insulators? And the best conductor turns out is diamond. Why? That's because the bumping, the spring.

So in this case is the wave, atomic wave that carries the heat. So if the bonding between the atoms is strong, the spring constant is large. The velocity is large. So that's why diamond is a good conductor. And metals are not that good. Metals are by electrons.

And insulators, you can see the wood, amorphous material, pores, those are poor thermal conductors because the atomic arrangement is random. And that randomness makes the wave harder to propagate in the material. So this is, again, macroscopic. Now I'm going to also give you the other view, microscopic of heat transfer, and the kinetic picture.

So what happens, how heat really is conducting? Let's think about the molecules. And the molecules, let's say we have a box. And one side is hot. The other side is cold. Let's forget about convection. How quick it goes from one [INAUDIBLE] side, we add the molecule to the cosine.

It's really on the hot side, the air molecule collided with the wall. And on the wall is hot. It means the velocity is faster. So this molecule will get a faster velocity due to that energy exchange between the wall and the air molecules. Now, this hot air molecule will collide with the [INAUDIBLE] molecule. Of course, we have a chance to go randomly.

Every molecule is moving a few 100 meters per second, as we talked before. And they collide with the cooler one. And they pump some of that excess energy to the next [? level. ?] The next [? level ?] will go back and forth. Again, they do this, cascade energy from the hot to the cold side. It's a diffusion process.

And typically, there is a the distance of this collision. You can go to estimate it. And just turns out that this distance between the average of multiple collision is about 100 nanometer. In this room, the air molecule distance between collision is about 100 nanometer.

And so now I give you a few more definition. The energy per particle, let's take energy per particle as e . And the average velocity is v . And time between collision is τ . And distance between collision is γ . That's the mean free path. So time is the average time between collision is the relaxation time. And this distance and the time is related by the velocity.

So what is a heat transfer? When I think about heat going through this imaginary surface here, how I calculate it? I can't. I do [INAUDIBLE]. I will say, OK, only those molecules within one mean free path. v_x is a velocity component in the x direction, times the collision time. So only those molecules can go zip through the interface without collision, because τ is the average time between collision.

So this part of the molecule is harder to go from this side to the other side. And on this side, the molecule is cooler again, in the v_x times τ [? average, ?] they go from this side to the other side. And I take the difference. I count all the molecules. I take the difference. That's my reflux. So let me write down the math. That's what I do.

I say on the left hand side, this is a per unit volume of how many. N is the particle number density, the second bullet there. Each particle, how much energy it has? And this is the velocity in the x direction. So I say only in this range, from x to $x + \Delta x$. In this range, you can go through. On the other side is $x + \Delta x$. You can do more detail. I hear this is a very simple argument.

So that gives me a [INAUDIBLE] for heat plotting. Go check a unit is a watt per meter squared, [INAUDIBLE] plus.

In the next lecture we'll just do a [INAUDIBLE] expansion of these two terms and write it into a differential form, because this is about 100 nanometers. If I do meters or centimeters or millimeters, that's essentially continuum, a very small Δx .

So I do the derivative. And I do a Taylor expansion. This one half is because there's only half of this go this way, half goes that way. So that's where the [INAUDIBLE] came from. And after I do the Taylor expansion, I have e and v and v_x . v_x is random velocity. I pull out this independent of distance. It could be dependent on this. I'm just making an assumption, approximation.

In there are different distance. This v_x^2 . v_x^2 is random. So it's one third of the average velocity, one third of v^2 . So that's my one third v^2 replacing this v_x^2 . Δx is here. And the n times e is a per unit volume how much energy, internal energy the molecule has.

So I have this n times e is u . And I take a general du/dx . OK, what you get? You got [INAUDIBLE]. Because here is one third v^2 , Δx [INAUDIBLE]. And that's the thermal conductivity. And this is the temperature gradient.

You can do that. You can go back and do your Fick's law and Newton's shear stress law. In fact, all the diffusion law you learn in three steps. But yes.

STUDENT: You said we could move the velocity out of the derivative because of its constant. But isn't the velocity related to the temperature?

GANG CHEN: Yeah, good question. This is later on when discuss the thermoelectric effect. I'll come back to this. Here I'm talking about a molecule. And I just wanted to show you the [? Fourier ?] law. And they're clearly approximation. And what are you talking is more like a coupled mass and heat transfer.

And, in fact, if you learn once the heat transfer, you know mass and heat transfer couples. So the thermal conductivity is one third specific heat velocity and mean free path. And the specific heat is per unit volume. If I do capital C_v per unit volume based, and typically we do per unit mass based since this is a density price per unit mass basis.

So this is the a very useful relation to keep in mind. Simple thermal conductivity depends on how many energy is stored by the molecule, how fast they travel, and how far they travel between quality. So if you want a high thermal conductivity, you want the power of [INAUDIBLE]. Low thermal conductivity, you reduce that.

And for most solid, they say is about 10 to the sixth power. It doesn't vary by a factor of 5. So when I do on my back of envelope order of magnitude estimation, I say is 10 to the sixth. You go to check. And velocity in solid is just you can roughly think the speed of sound, few 100 or few 1,000 meters per second. The mean free path is short-- nanometers, 100 nanometers. So that's the range order.

So this is I think most of you, probably in 2005, you haven't seen it. It's very simple. But the picture is really the diffusion picture or similar. Let me give you-- I haven't finished radiation. So I come back to-- I said that Stefan-Boltzmann law.

And what I want to show you next is how I can derive Stefan-Boltzmann law from the statistics. I just showed. So I'm considering enclosure. Let me imagine the enclosure is perfectly reflecting. There are photons. And the walls is the temperature, T . So it's a equilibrium system. I want to see how many the electromagnetic waves inside this box, how much energy they have, a constant temperature-- equilibrium, thermal equilibrium.

So let's go back to a little bit of the basic relations of thermal electromagnetic wave. We know the speed of light is proportional to its speed of light as a wavelength times frequency. That's a high school learn. And definition, I say angular velocity is $2\pi\mu$. We vector magnitude is 2π over λ . What is the wave vector? The wave propagates in certain direction.

And so because the direction is a vector. $2\pi/\lambda$, λ is a wavelength. It's a scalar. And the rationale is the vector. So it's a k_x, k_y, k_z if you do Cartesian coordinate. And so I can write it C equals μ times λ . In terms of frequency and vector, it's c times ω equals to ct . So this relation between the vector, magnitude, and the frequency is called dispersion.

And in the speed-- in the case of light, it's a linear. C is 310 to the eighth meter per second. And, of course, because the vector, so it's a square root of the components. So that's the dispersion. And now I want to count. My question is, how much energy photon has inside this box? I'm going to count how many modes you have. Remember each mode, the Bose-Einstein distribution already tells me for each mode how many photons.

So how many modes? So that's basically I'm counting. Because I'm a perfect [INAUDIBLE], you can see the mode has to be standing wave here, either this way or this way. That boundary is 0.

STUDENT: Why is it 0? Why should it be 0?

GANG CHEN: Otherwise it doesn't get a steady state. It keeps propagating. So this is my wavelength component in the x direction. k_x is a corresponding wavelength in x direction. And the L_x is the length here. So if I just say, this must satisfy the rule, then I have to have integer. Those are the possibilities.

And that translated in the possibility of my k . k_x just is 2π over λ_x . So this is $n\pi/L_x$. OK, so each combination of x and y and z , each set gives me one mode. And I want to find out how much energy, I just sum up all those modes. So I sum that up.

And remember, when I do molecules, I do velocity from minus infinity to plus infinity. And here when I do this, I do discrete summation of n_x , and n_y , and n_z . So I do this summation. This is an x, y , and z . Each mode has this is the number of photons in this area set. And this is the energy of a photon is [INAUDIBLE]. I got the factor of 2 because the electromagnetic wave is a transverse wave. And it has a two component, two polarizations, so that factor of 2 [INAUDIBLE].

OK, if you don't look at this, that's where my starting point. And what I want to do is, rather than do the summation, I'm going to translate it into integration. Summation is over n .

So if you look at this interval between n and k , it's $2\pi Lx$, so when I have a dkx , I divide by $2\pi Lx$. That's n if this is large number. If small, you have a lot of problem.

So I translate this summation, n_x , into this dkx integration. I do same for k_y , k_z . This is-- I'm just copying. I go the extra step here just for the later purpose. Because my function is a is even function. So I extend to minus infinity to plus infinity. The difference is the factor of 2. So I don't have this 2 here anymore, because the factor of 2 is gone. That's just for my later purpose.

The end result, this is just an equivalent. And what it means is I can, rather than standing wave, I can say do two counter-propagating. One goes left, one goes right. That's when you do the maximal velocity is minus infinity, plus infinity. So that's equivalent.

OK, I think I will probably have to stop here. There are two more slides you can look at, but I'll discuss. What's coming is the blackbody radiation in each wavelength, how much energy varies with simple [INAUDIBLE]. OK.